A Two-Dimensional Predator-Prey Model

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ABSTRACT

A spatio-temporal predator-prey model is devised from an existing space-independent model that was modified to include the phenomenon of migration. A tolerance level, which depends on the amount of resources, is proposed as the motivation for the migration of the prey. The change from a rectangular to a cylindrical confinement was interpreted as an increase in mobility. The model supports the idea of increased predator population with mobility.

Key words: predator-prey model, migration, mobility, spatio-temporal, numerical

INTRODUCTION

This work was motivated by the lack of literature on spatio-temporal predator-prey models. The current effort is an initial step in identifying the parameters that lead to a particular community structure (Cunningham & Saigo, 1995), which is dependent on the spatial distribution of resources, and may be relevant in the study of the emergence of patchy environments (Shorrocks & Swingland, 1990).

Most of the existing predator-prey models are described by Lotka-Volterra type relations. We also use one (Murray, 1993) as the basis of our model.

$$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right)-NPR\,(N) \qquad (1)$$

$$\frac{dP}{dt} = -d_0P + ePR\,(N) \qquad (2)$$

Here, N denotes the prey, and P the predator population. The other parameters are K-the carrying capacity, d_0-the death rate of predators, r-the response time of prey, and e-an enhancement factor for the predator response. The predator response $R(N)$ defines saturation for prey-intake. We take this to have the following form

$$R(N) = \frac{A}{N+B} \qquad (3)$$

THE MODEL

Equations (1) and (2) are integrated using Euler's method with a step size of unity, effectively making the model discrete. The resulting equations, taken to apply locally, give the current populations in terms of the immediately preceding populations. The spatial dynamics is effected by imposing a local population tolerance level for both predator and prey. Once the tolerance level is reached, the excess population relocates itself.

The prey migrates as soon as the resulting population reaches a certain fraction of the carrying capacity, K. The preferred location for migration is defined as that which contains the least amount of the sum of predators and prey. It is interpreted here that the presence of either the predator or the prey in a community reduces the amount of resources that benefit the prey.

The effective resource for the predator is the amount of prey in the community. The predator is taken to migrate when a fraction of the total prey population in the community drops below its own. The migrating predators seek the location with the most number of preys.

The effective local discrete model is then given by the following equations.

$$N_{t+1} = N_t + rN_t\left(1-\frac{N_t}{K}\right) - N_tP_tR(N_t), \qquad N_{t+1} \le tolN \qquad (4)$$

$$= tolN, \qquad N_{t+1} > tolN$$

$$P_{t+1} = P_t - d_0P_t + eP_tR(N_t), \qquad P_{t+1} \le tolP \qquad (5)$$

$$= tolP, \qquad P_{t+1} > tolP$$

Here *tolN* and *tolP* are the prey and predator local population tolerances. The second equality in Equations (4) and (5) apply when the right-hand side of the first equation exceeds the tolerance levels. The population in excess of these tolerances relocates itself in accordance with our given rules.

The above were implemented in a 3x3 grid. Movement of predator and prey is allowed only in four adjacent locations. If we label the current location *(i,j)*, where *i* and *j* label row and column, respectively, the allowed migration locations are *(i, j-1)*, *(i, j+1)*, *(i-1,j)*, and *(i+1,j)*. This is also the sequence at which the locations are considered for relocation assuming equally optimal conditions for each location. The search algorithm was implemented with the aid of standard indexing procedures (Press et al., 1989) in the sorting of the intermediate arrays for the population distribution of prey and predator.

Two types of confinement were applied to the populations. A cylindrical confinement allows the movement from the leftmost edge location of the grid to the rightmost edge location, while rectangular confinement prohibits movement beyond the edges of the two-dimensional array. Taking the rectangular constraint as the base, we have an increase in the average number of possible relocation positions in a cylindrical confinement. This is made apparent by considering a population located at any corner of the two-dimensional array. The population has only two possible relocation sites if it were constrained to the rectangular array, but it would have three if it were confined to a cylinder. This difference is interpreted as an increase in the mobility of both predator and prey.

RESULTS AND DISCUSSION

The tolerance levels for both predator and prey were set to one-tenth of their perceived resources: $N/10$ for the predator and $K/10$ for the prey. The world was set to have uniform carrying capacity throughout, with $K = 2{,}000$. Other parameters were set as follows: $r = 1.0$, $A = 3$, $B = 100$, $d_0 = 0.8$, and $e = 100$.

The initial conditions were chosen such that the local predator and prey populations achieve a steady state value in the absence of population movements into the community. The steady state values are equal to their respective local tolerances. The initial populations were located at the central grid position with values 133 and 2 for prey and predator, respectively.

The resulting total population for prey and predator in nine communities are shown in Figs. 1 and 2. There is a marked difference between the total population for both predator and prey after about 15 steps in the two confinements considered here. The populations constrained to move in a rectangular region seem to achieve steady state values while the cylindrically constrained populations have yet to show any clear indications of saturation.

There is, however, a clear indication of increased total population for predators in cylindrical confinement against those in a rectangular boundary. The resulting spatial population distribution for predators, rounded to integer values, after 20 steps are given below.

rectangular confinement			cylindrical confinement		
0	0	0	0	15	0
0	32	17	16	32	8
0	10	0	0	19	0

The cylindrically confined predators were able to occupy more communities after 20 steps, the time for

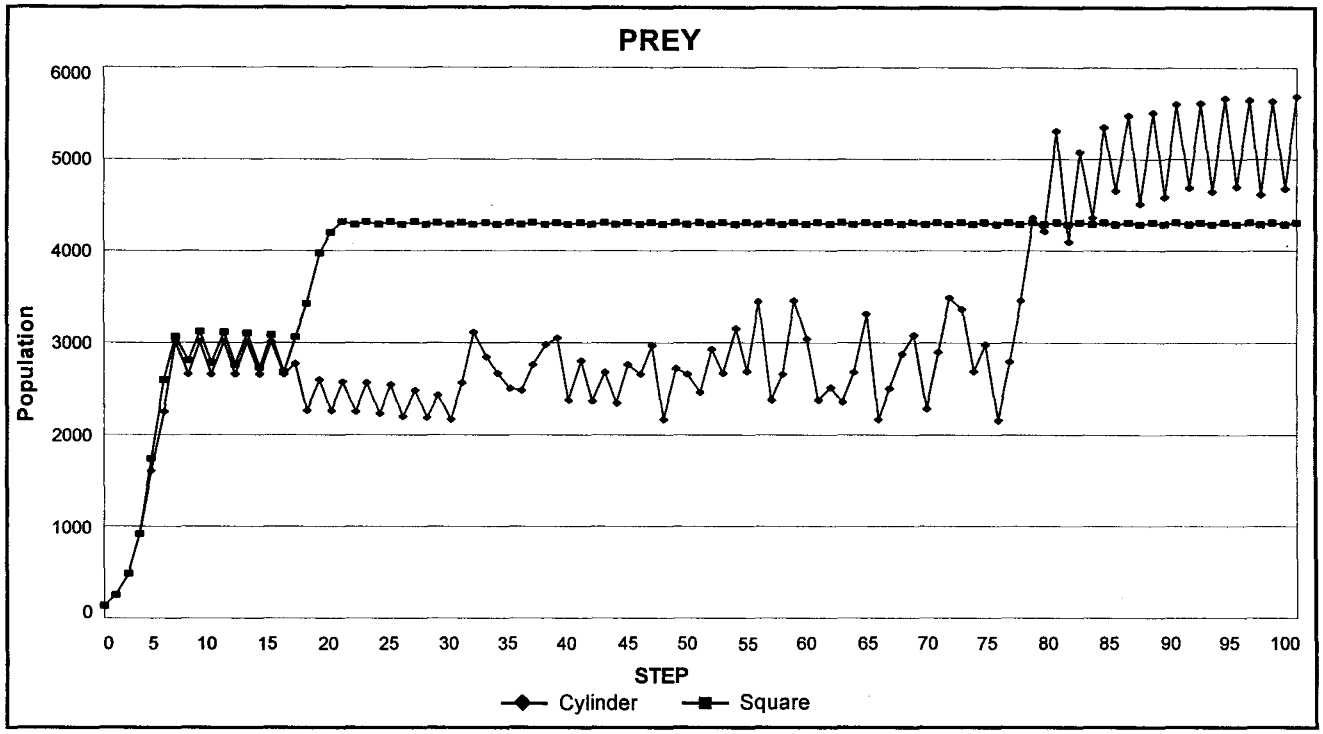

Fig. 1. Total prey population in nine communities for cylindrical and rectangular confinement. Initial conditions were N = 133, P = 2

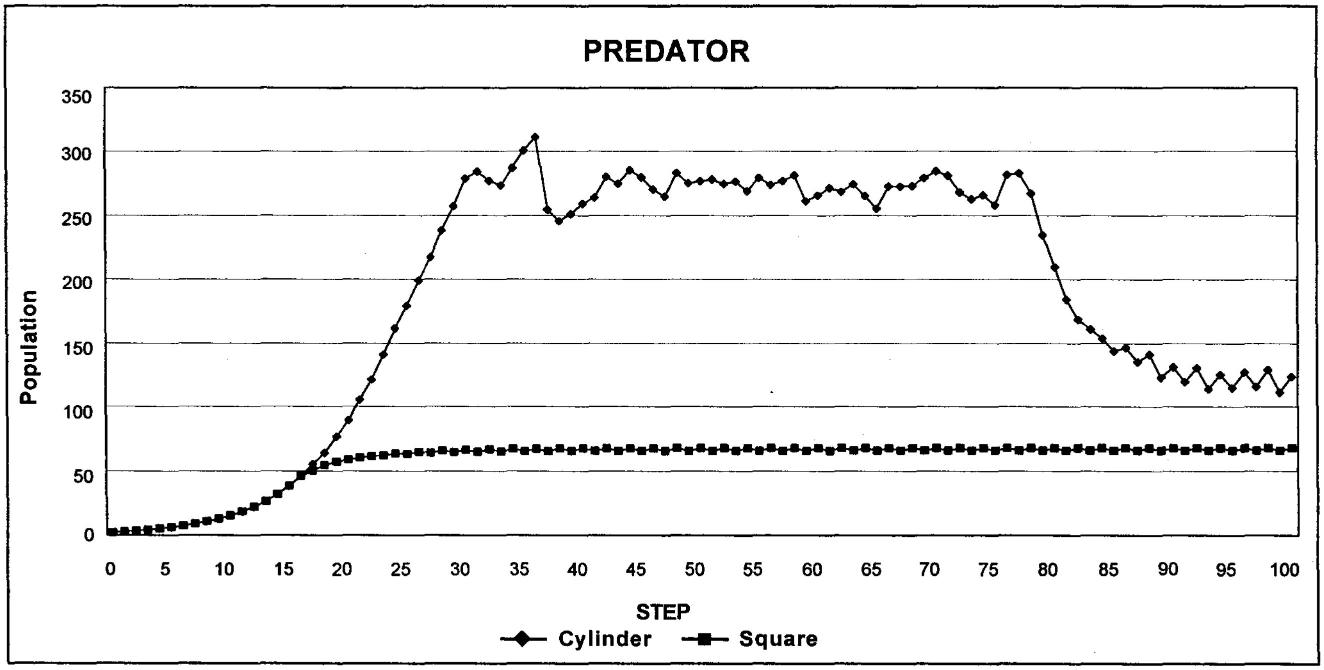

Fig. 2. Total predator population in nine communities for cylindrical and rectangular confinement. Initial conditions were N = 133, P = 2

which rectangular confinement of predators seem to saturate, compared to those constrained to a rectangular region. The increased mobility for predators in cylindrical confinement proved beneficial in the sense that they were able to achieve larger populations and occupy more communities. The model supports the idea that increased mobility is an advantageous characteristic for predators.

rectangular confinement			cylindrical confinement		
200	200	352	200	200	367
317	200	688	200	200	317
1964	200	200	688	200	200

The resulting spatial population distribution for preys, rounded to integer values, after 20 steps are given below.

All nine communities are occupied in both types of confinement. The smaller populations in cylindrically confined preys are results of the greater number of predators. The preys constrained to move in a rectangular region may have a similar spatial distribution after longer time steps. As shown in Fig. 1, the total prey population does not increase at later times. It is difficult to account for the spatial distribution of the cylindrically confined preys. Fig. 1 shows a greater total population for them compared to preys in rectangular confinement after about 80 steps.

A different set of initial conditions (N = 60; P = 5) did not show clear saturation values for either boundary condition for prey and predator populations. However, it also resulted in increased total population for predators in cylindrical confinement, again indicating the benefits of increased mobility.

It is hoped that this initial investigation will yield a model that accounts for the emergence of particular community structures, a field of much interest to environmentalists. This will necessarily entail a larger grid for the spatial representation, and the effects of the boundary conditions are not expected to be significant for short times. It is also our intention to systematically analyze the stability of similar spatial dynamical systems in the future.

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REFERENCES

Cunningham, W.P. & B.W. Saigo, 1995. Environmental Science, 3rd ed. Dubuque, IA, Bogota, Boston, Buenos Aires, Caracas, Chicago, Guilford, CT, London, Madrid, Mexico City, Sydney, Toronto, Wm. C. Brown: 64-89 pp.

Murray, J.D., 1993. Mathematical Biology, 2nd ed. Berlin, Heidelberg, New York, Springer: 70-72 pp.

Press, W.H. et al., 1989. Numerical Recipes. Cambridge, New York, Port Chester, Melbourne, Sydney, Cambridge: 232-234 pp.

Shorrocks, B. & I.R. Swingland (eds.), 1990. Living in a Patchy Environment. Oxford, New York, Tokyo, Oxford: