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Domination in $m-$ polar soft fuzzy graphs

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Abstract

In this article, we have introduced dominating set, minimal dominating set, independent dominating set, maximal independent dominating set in m polar soft fuzzy graphs. We proved theorems and also some properties of dominating set in m polar soft fuzzy graphs

Keywords: Dominating set, Independent dominating set, Maximal independent dominating set in $m-$ polar soft fuzzy graphs.
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1 Introduction

The work of Zadeh [13] in 1975, which interacted with ambiguity and imprecision between absolute true and absolute false, is credited with giving rise to fuzzy set theory. A fuzzy set's possible values fall between [0,1]. Fuzzy set theory's astonishing discovery opened up new possibilities for handling uncertainty in a variety of scientific and technological fields. Due to its use in engineering, communication networks, computer sciences, and artificial intelligence, graph theory is swiftly becoming a mainstream topic in mathematics.Graphs and fuzzy graphs are investigated in [10, 11].The idea of domination in fuzzy graphs was propounded by A. Somasundaram and S. Somasundaram [14] in 1998.Soft set and hybrid models are used to deal with uncertainty based on parametrization tool. Soft set, fuzzy soft set and m-polar fuzzy sets are studied in [1, 4, 6, 7]. The possibilityof domination in $m-$ polar fuzzy graphs was introduced by M. Akram et.al [2] in 2017. Domination in graphs and domination in fuzzy graphs are analysedin [9, 5]. MohintaSumit and Samanta.T.K. [8] proposed the thought of fuzzy soft graph. S. Ramkumar and R. Sridevi [12] presented their perception in $m-$polar soft fuzzy graphs. These concepts motivate us to introduce domination in$m-$ polar soft fuzzy graphs.

2 Dominating set, Minimal dominating set, Maximal independent set in $m-$ Polar soft fuzzy graphs

In this paper m-psf-graph denote $m-$ polar soft fuzzy graph.

Definition 2.1. *An edge in an m-psf-graph $\widetilde{G}_{P,V} = (G^*, \widetilde{\rho}, \widetilde{\mu}, P)$is defined as an effective edge if*

$$\begin{aligned}
&\widetilde{\mu}_e x_1(uv) = (\widetilde{\rho}_e x_1(u) \wedge \widetilde{\rho}_e x_1(v))\\
&\widetilde{\mu}_e x_2(uv) = (\widetilde{\rho}_e x_2(u) \wedge \widetilde{\rho}_e x_2(v))\\
&\dots\dots\dots\dots\dots\dots\\
&\dots\dots\dots\dots\dots\dots\\
&\widetilde{\mu}_e x_m(uv) = (\widetilde{\rho}_e x_m(u) \wedge \widetilde{\rho}_e x_m(v))
\end{aligned}$$

in $N^{hd}(u) = \{v \in V/v\ is\ a\ neighbor\ of\ u\}$ is called the neighbourhood of u.

Definition 2.2. *A vertex $u \in V$ in an m-psf-graph $\widetilde{G}_{P,V} = (G^*, \widetilde{\rho}, \widetilde{\mu}, P)$ is said to be an isolated vertex if*

$$\begin{aligned}
&\widetilde{\mu}_e x_1(uv) < (\widetilde{\rho}_e x_1(u) \wedge \widetilde{\rho}_e x_1(v))\\
&\widetilde{\mu}_e x_2(uv) < \ (\widetilde{\rho}_e x_2(u) \wedge \widetilde{\rho}_e x_2(v))\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&\widetilde{\mu}_e x_m(uv) < (\widetilde{\rho}_e x_m(u) \wedge \widetilde{\rho}_e x_m(v)) \forall v \in V \setminus \{u\}
\end{aligned}$$

in $\widetilde{H}_{P,V}(e)\ \forall\ e \in P$*. so that* $N^{hd}(u) = \phi$.

Example 2.1. *Consider a* $3 - psf - graph \widetilde{G}_{P,V}$.

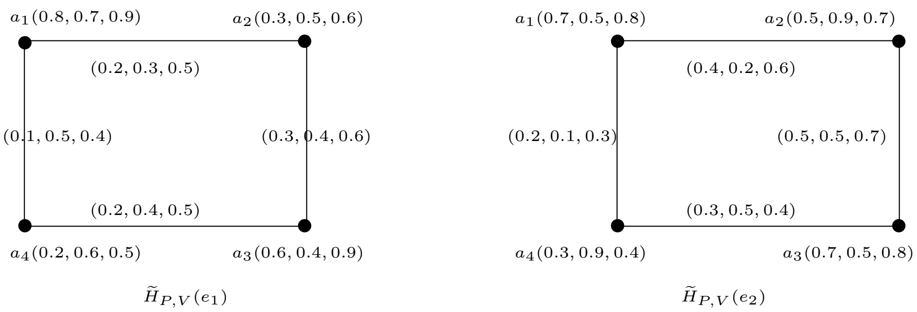

Figure.1. $3-$ psf-graph $\widetilde{G}_{P,V} = \{\widetilde{H}_{P,V}(e_1), \widetilde{H}_{P,V}(e_2)\}$.

In this example, a_2a_3 and a_3a_4 are effective edges.
Also $N^{hd}(a_1) = \{\phi\}, N^{hd}(a_2) = \{a_3\}, N^{hd}(a_3) = \{a_4a_2\}, N^{hd}(a_4) = \{a_3\}$.
Here a_1 is an isolated vertex.

Definition 2.3. *Let* $\widetilde{G}_{P,V} = (G^*, \widetilde{\rho}, \widetilde{\mu}, P)$ *be an* m*-psf-graph. For any two vertices* $u, v \in V$*, we call* u *dominates* v *in* $\widetilde{G}_{P,V}$ *if*

$$\begin{aligned}
&\widetilde{\mu}_e x_1(uv) = (\widetilde{\rho}_e x_1(u) \wedge \widetilde{\rho}_e x_1(v))\\
&\widetilde{\mu}_e x_2(uv) = (\widetilde{\rho}_e x_2(u) \wedge \widetilde{\rho}_e x_2(v))\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&\widetilde{\mu}_e x_m(uv) = (\widetilde{\rho}_e x_m(u) \wedge \widetilde{\rho}_e x_m(v))
\end{aligned}$$

in $\widetilde{H}_{P,V}(e)\ \forall e \in P$ *and* $\forall u, v \in V$.

Definition 2.4. *A subset* $\widetilde{\mathcal{S}}$ *of* V *be an* $m-$ *psf-graph* $\widetilde{G}_{P,V}$*. Then cardinality of* $\widetilde{\mathcal{S}}$ *is defined as,*

$$|\ \widetilde{\mathcal{S}}\ |= \sum_{e\in P}(\sum_{u\in\widetilde{\mathcal{S}}} \widetilde{\rho}_e(u))\ in\ \widetilde{H}_{P,V}(e)p.$$

Definition 2.5. *A subset $\widetilde{\mathcal{S}}$ of V is called a dominating set of an $m-psf-graph\widetilde{G}_{P,V}$ if for every vertex $u \in V \setminus \widetilde{\mathcal{S}}$ then $\exists\, v \in \widetilde{\mathcal{S}}$ such that u dominates v in $\widetilde{H}_{P,V}(e)$ $\forall e \in P$.The domination number $\gamma(\widetilde{G}_{P,V})$ means the infimum cardinality of all dominating set in $\widetilde{G}_{P,V}$ and $\gamma(\widetilde{G}_{P,V}) = min_{\widetilde{S}\in V} \sum_{e\in P}(\sum_{u\in\widetilde{S}} \widetilde{\rho}_e(u))$.*

Definition 2.6. *A dominating set $\widetilde{\mathcal{S}}$ is called a minimal dominating set of $m-psf-graph\widetilde{G}_{P,V} = (G^*, \widetilde{\rho}, \widetilde{\mu}, P)$ if for any $a \in \widetilde{\mathcal{S}}, \widetilde{\mathcal{S}} \setminus \{a\}$ is not a dominating set in $\widetilde{H}_{P,V}(e)$ $\forall\, e \in P$.*

Definition 2.7. *Lower and upper domination number of an $m-$psf-graph $\widetilde{G}_{P,V} = (G^*, \widetilde{\rho}, \widetilde{\mu}, P)$ is denoted by $\gamma(\widetilde{G}_{P,V})$ and $\Gamma(\widetilde{G}_{P,V})$, respectively, and defined by infimum cardinality and supremum cardinality of all minimal dominating set of that $m-$psf-graph, respectively.*

Example 2.2. *Consider a $3-$psf-graph $\widetilde{G}_{P,V}$.*

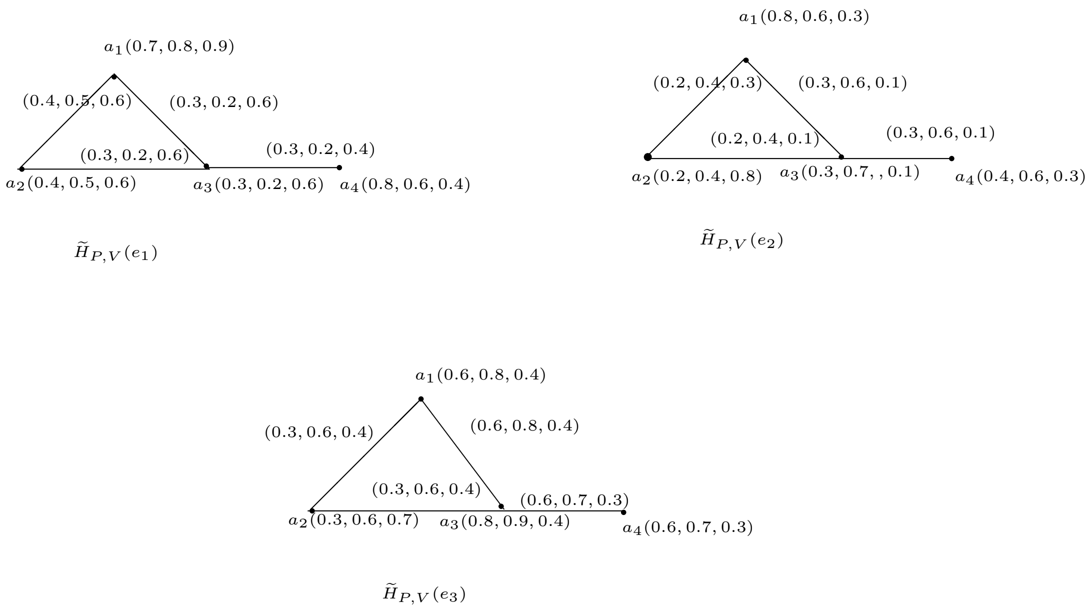

Figure.2. Minimal dominating set of 3-psf-graph

In Figure.2. Here, the minimal dominating sets in each parameterized graph is

$$\widetilde{\mathcal{S}}_1 = \{a_1, a_4\}, \widetilde{\mathcal{S}}_2 = \{a_2, a_4\}, \widetilde{\mathcal{S}}_3 = \{a_3\}.$$

Theorem 2.1. *A dominating set $\widetilde{\mathcal{S}}$ is minimal if and only if one of the below mentioned criteria holds.*

1. *$N^{hd}(a) \cap \widetilde{S} = \phi$.*
2. *There is a vertex $b \in V \setminus \widetilde{S}$ such that $N^{hd}(b) \cap \widetilde{S} = \{a\}$, for each $a \in \widetilde{S}$.*

Proof. For a minimal dominating set $\widetilde{S}$ of a $3-$psf-graph $\widetilde{G}_{P,V}$, for every vertex $a \in \widetilde{S}, \widetilde{S} \setminus \{a\}$ is not dominating set and so then $\exists\, b \in V \setminus (\widetilde{S} \setminus \{a\})$ which is not dominated by any vertex in $\widetilde{S} \setminus \{a\}$ of $\widetilde{H}_{P,V}(e_i)\ \forall\ e_i \in P$ for $i = 1, 2, \ldots, n$. If $a = b$ then $N^{hd}(a) \subseteq V \setminus \widetilde{S} \Rightarrow N^{hd}(a) \cap \widetilde{S} = \phi$ in $\widetilde{H}_{P,V}(e_i)$ for $i = 1, 2, \ldots, n$. If $a \neq b$, then b is not dominated by $\widetilde{S} \setminus \{a\}$ but is dominated by $\widetilde{S}$, i.e., b is dominated only by a in $\widetilde{S}$ in $\widetilde{H}_{P,V}(e_i)\ \forall\ e_i \in P$ for $i = 1, 2, \ldots, n$. $\therefore N^{hd}(b) \cap \widetilde{S} = \{a\}$ in $\widetilde{H}_{P,V}(e_i)\ \forall\ e_i \in P$ for $i = 1, 2, \ldots, n$. Hence $N^{hd}(b) \cap \widetilde{S} = \{a\}$ in $3-$psf-graph $\widetilde{G}_{P,V}$.

Conversely, let $\widetilde{S}$ holds one of the two given criterias. If $\widetilde{S}$ is not minimal dominating set in $\widetilde{H}_{P,V}(e_i)\ \forall\ e_i \in P$ for $i = 1, 2, \ldots, n$. Then $\exists$ a vertex $a \in \widetilde{S}$ such that $\widetilde{S} \setminus \{a\}$ is a dominating set in $\widetilde{H}_{P,V}(e_i)$ for $i = 1, 2, \ldots, n$. Thus a is dominated by atleast one vertex in $\widetilde{S} \setminus \{a\}$ in $\widetilde{H}_{P,V}(e_i)$ for $i = 1, 2, \ldots, n$. Then $N^{hd}(a) \nsubseteq \widetilde{S} \setminus \{a\}$ in $\widetilde{H}_{P,V}(e_i)$ for $i = 1, 2, \ldots, n$. Hence condition (1) does not hold. Also, If $\widetilde{S} \setminus \{a\}$ is a dominating set in $\widetilde{H}_{P,V}(e_i)$ for $i = 1, 2, \ldots, n$. Then every vertex b in $V \setminus (\widetilde{S} \setminus \{a\})$ is dominated by at least one vertex in $\widetilde{S} \setminus \{a\}$ in $\widetilde{H}_{P,V}(e_i)$ for $i = 1, 2, \ldots, n$. So $N^{hd}(b) \cap \widetilde{S} \neq \{a\}$ in $\widetilde{H}_{P,V}(e_i)$ for $i = 1, 2, \ldots, n$. hence condition (2) does not hold. This leads to a $\Rightarrow\Leftarrow$. $\therefore \widetilde{S}$ must be a minimal dominating set in $\widetilde{H}_{P,V}(e_i)$ for $i = 1, 2, \ldots, n$. Hence $\widetilde{S}$ must be a minimal dominating set in $3-$ psf-graph $\widetilde{G}_{P,V}$. □

Theorem 2.2. *Let $\widetilde{G}_{P,V} = ((P, \widetilde{\rho}), (P, \widetilde{\mu}))$ be a $3-$psf-graph without isolated vertices. If $\widetilde{S}$ is a minimal dominating set of $\widetilde{G}_{P,V}$ then $V \setminus \widetilde{S}$ is a dominating set of $\widetilde{G}_{P,V}$.*

Proof. Let $\widetilde{S}$ be a minimal dominating set and $a \in \widetilde{S}$. Since $\widetilde{G}_{P,V}$has no isolated vertices $\exists$ a vertex $b \in N^{hd}(a)$ in $\widetilde{H}_{P,V}(e_i)\ \forall\ e_i \in P$ for $i = 1, 2, \ldots, n$. Utilization of the same content similar to the proof given for Theorem 2.1, we get that b in $V \setminus \widetilde{S}$. Thus every element of $\widetilde{S}$ is dominated by some element of $V \setminus \widetilde{S}$ in $\widetilde{H}_{P,V}(e_i)$ $\forall\ e_i \in P$ for $i = 1, 2, \ldots, n$. Consequently $V \setminus \widetilde{S}$ is a dominating set of $\widetilde{G}_{P,V}$. □

Theorem 2.3. *Superset of a dominating set in $\widetilde{G}_{P,V}$ is a dominating set.*

Proof. Proof is obvious. □

Remark 2.1. *Subset of a dominating set in $\widetilde{G}_{P,V}$ need not to be dominating set.*

Example 2.3. *Consider a $3-$psf-graph.*

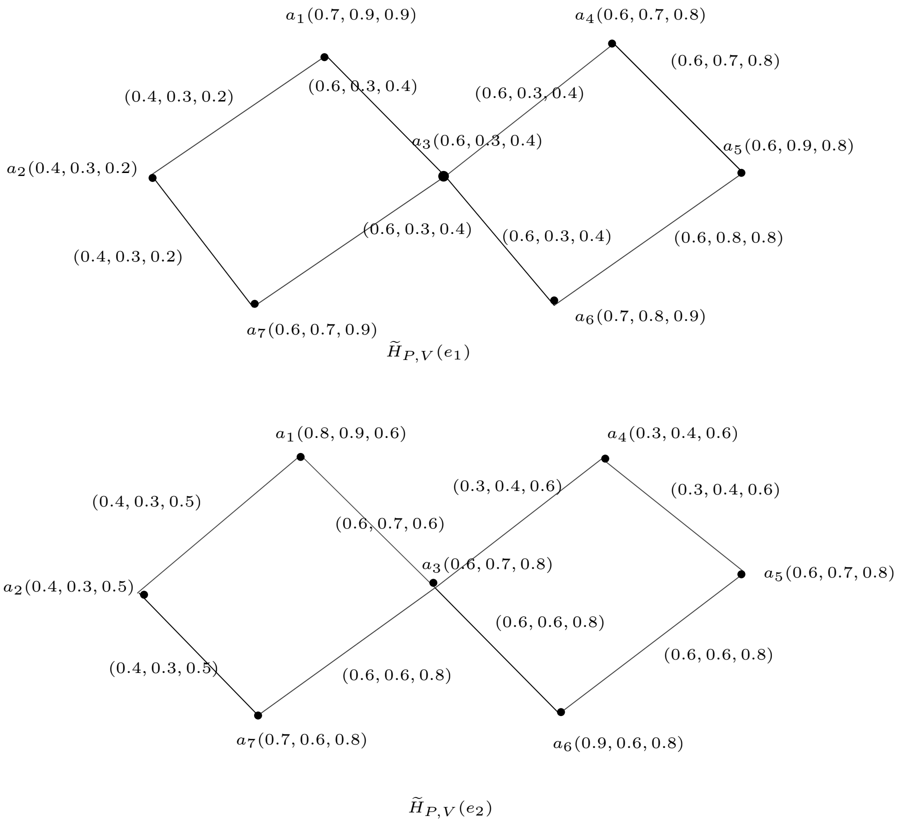

Figure.3. 3-psf-graphs.

Here each parameterized graph is $\widetilde{\mathcal{S}} = \{a_2, a_3, a_5\}$ and $\widetilde{\mathcal{S}} \cup \{b\} = \{a_1, a_2, a_3, a_5\}$ $\widetilde{\mathcal{S}} \setminus \{b\} = \{a_2, a_5\}$. Here $\{a_1, a_2, a_3, a_5\}$ is a dominating set. But $\{a_2, a_5\}$ is not a dominating set.

Definition 2.8. *A set $\widetilde{\mathcal{S}} \subseteq V$ of an $m-$psf-graph $\widetilde{G}_{P,V}$ is called an independent set if*

$\widetilde{\mu}_e x_1(uv) < (\widetilde{\rho}_e x_1(u) \wedge \widetilde{\rho}_e x_1(v))$
$\widetilde{\mu}_e x_2(uv) < (\widetilde{\rho}_e x_2(u) \wedge \widetilde{\rho}_e x_2(v))$
$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$
$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$
$\widetilde{\mu}_e x_m(uv) < (\widetilde{\rho}_e x_m(u) \wedge \widetilde{\rho}_e x_m(v))$ *in $\widetilde{H}_{P,V}(e)\ \forall\ e \in P$ and for all $u, v \in \widetilde{\mathcal{S}}$.*

Definition 2.9. *An independent set $\widetilde{\mathcal{S}}$ of an $m-$ psf-graph $\widetilde{G}_{P,V} = (G^*, \widetilde{\rho}, \widetilde{\mu}, P)$is said to be maximal independent set if for every vertex $u \in V \setminus \widetilde{\mathcal{S}}$, the set $\widetilde{\mathcal{S}} \cup \{u\}$ is not independent in $\widetilde{H}_{P,V}(e)\ \forall\ e \in P$.*

Definition 2.10. *Lower and upper independence number of an $m-psf-graph$ $\widetilde{G}_{P,V} = (G^*, \widetilde{\rho}, \widetilde{\mu}, P)$ is denoted by $i(\widetilde{G}_{P,V})$ and $I(\widetilde{G}_{P,V})$, respectively, and defined by infinimum cardinality and supremum cardinality among all the maximum independent set of that $m-$psf-graph, respectively.*

Example 2.4. *Consider a $3-$psf-graph $\widetilde{G}_{P,V}$.*

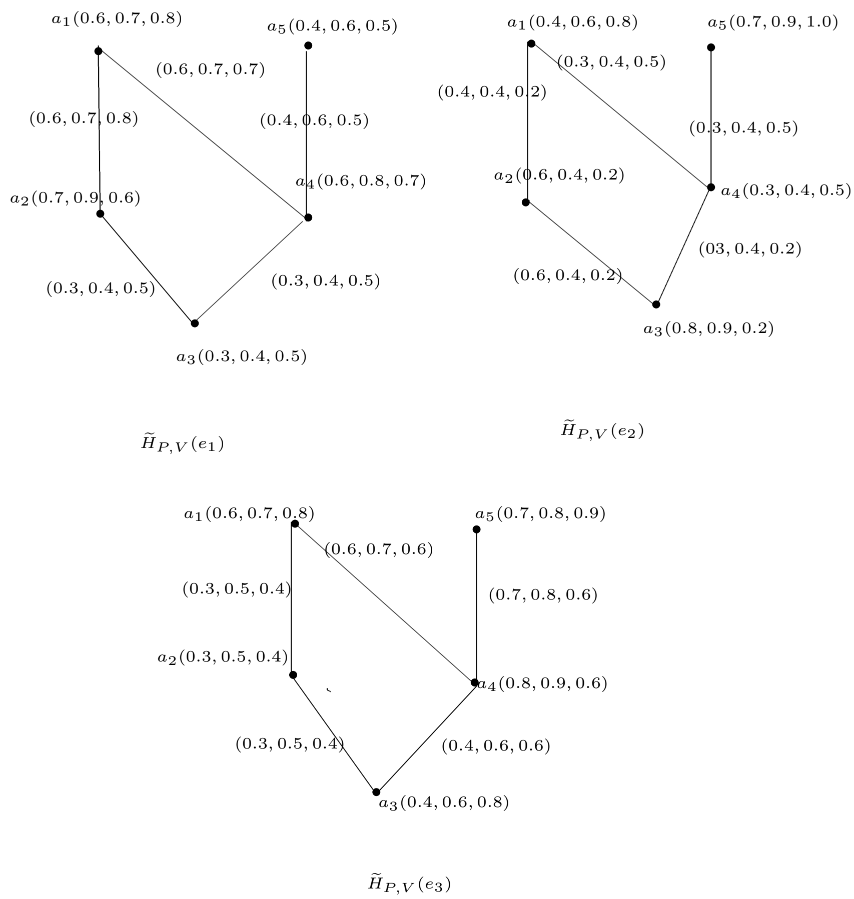

Figure.4. Independent set of a 3-psf-graphs.

In this example, the dominating set in each parameterized graph is $\{\{a_3, a_4\}, \{a_1, a_3, a_5\}, \{a_2, a_4\}\}$. In $\widetilde{H}_{P,V}(e_i)\ \forall\ e_i \in P$ for $i = 1, 2, 3$. the maximal independent set is $\{\{a_2, a_4\}, \{a_1, a_3, a_5\}\}$. Here $\{a_2, a_4\}$ is a maximal independent set of $\widetilde{G}_{P,V}$ with infimum cardinality and $i(\widetilde{G}_{P,V}) = (3.3, 3.9, 3.0)$. Also here $\{a_1, a_3, a_5\}$ is maximal independent set of $\widetilde{G}_{P,V}$ with supremum cardinality and $I(\widetilde{G}_{P,V}) = (4.9, 6.2, 6.7)$.

Proposition 2.1. *For any* $3-psf-graph$ $\widetilde{G}_{P,V} = (G^*, \widetilde{\rho}, \widetilde{\mu}, P), \gamma(\widetilde{G}_{P,V}) \leq i(\widetilde{G}_{P,V})$.

Example 2.5. *Consider a* $3-$ *psf-graph.*

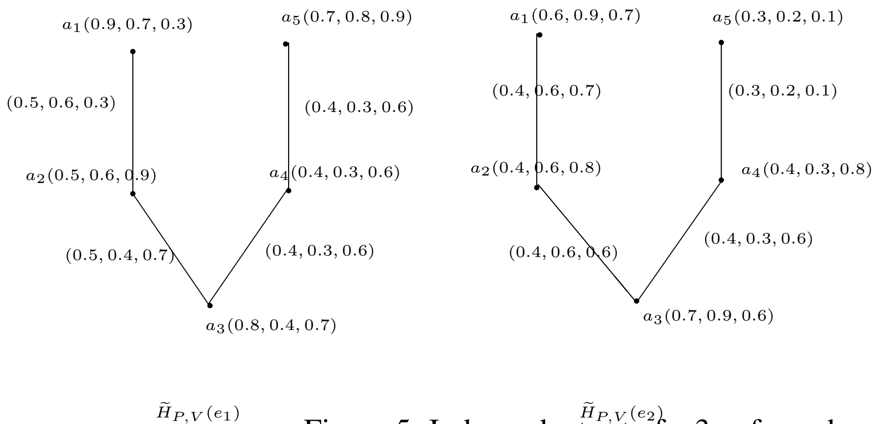

Figure.5. Independent set of a 3-psf-graphs

In Fig.5. that minimum dominating set of a $3-$psf-graph $\widetilde{G}_{P,V}$ is $\{a_2, a_4\}$ and the maximal independent set of $\widetilde{G}_{P,V}$ is $\{a_1, a_3, a_5\}$ in $\widetilde{H}_{P,V}(e_i)$ $\forall$ $e \in P$ for $i = 1, 2$. Also here $\gamma(\widetilde{G}_{P,V}) = (1.7, 1.8, 3.1)$ and $i(\widetilde{G}_{P,V}) = (4.0, 3.9, 3.3)$ in $\widetilde{H}_{P,V}(e_i)$ $\forall$ $e \in P$ for $i = 1, 2$. Clearly, $\gamma(\widetilde{G}_{P,V}) \leq i(\widetilde{G}_{P,V})$.

Theorem 2.4. *A set* $\widetilde{\mathcal{S}} \subseteq V$ *is a maximal independent set of a* $3-psf-graph$ $\widetilde{G}_{P,V}$ *if and only if it is independent and dominating set.*

Proof. Assume that $\widetilde{\mathcal{S}}$ is a maximal independent set of $\widetilde{G}_{P,V}$. Then for each vertex $a \in V \setminus \widetilde{\mathcal{S}}$, the set $\widetilde{\mathcal{S}} \cup \{a\}$ is not independent set in $\widetilde{H}_{P,V}(e_i)$ for $i = 1, 2, \ldots, n$. In this manner for every vertex $a \in V \setminus \widetilde{\mathcal{S}}$,then $\exists$ vertex $b \in \widetilde{\mathcal{S}}$ such that b dominates a in $\widetilde{H}_{P,V}(e_i)$ $\forall$ $e_i \in P$ for $i = 1, 2, \ldots, n$. Hence $\widetilde{\mathcal{S}}$ is a dominating set in $\widetilde{H}_{P,V}(e_i)$ $\forall$ $e_i \in P$ for $i = 1, 2, \ldots, n$. Therefore $\widetilde{\mathcal{S}}$ is both independent and dominating set in $\widetilde{H}_{P,V}(e_i)$ $\forall$ $e_i \in P$ for $i = 1, 2, \ldots, n$. Hence $\widetilde{\mathcal{S}}$ is both independent and dominating set in $\widetilde{G}_{P,V}$

Conversely if we suppose that $\widetilde{\mathcal{S}}$ is not maximal independent set in $\widetilde{H}_{P,V}(e_i)$ $\forall$ $e_i \in P$ for $i = 1, 2, \ldots, n$. Then $\exists$ a vertex $a \in V \setminus \widetilde{\mathcal{S}}$, such that $\widetilde{\mathcal{S}} \cup \{a\}$ is independent set. Thus there $\nexists$ any vertex b in $\widetilde{\mathcal{S}}$ which dominates a in $\widetilde{H}_{P,V}(e_i)$ for $i = 1, 2, \ldots, n$. So $\widetilde{\mathcal{S}}$ is not a dominating set in $\widetilde{H}_{P,V}(e_i)$ $\forall$ $e_i \in P$ for $i = 1, 2, \ldots, n$. Which $\Rightarrow\Leftarrow$ to the choice of $\widetilde{\mathcal{S}}$. Accordingly $\widetilde{\mathcal{S}}$ is a maximal independent set in $\widetilde{H}_{P,V}(e_i)$ $\forall$ $e_i \in P$ for $i = 1, 2, \ldots, n$. Hence $\widetilde{\mathcal{S}}$ is a maximal independent set of $\widetilde{G}_{P,V}$

□

Theorem 2.5. *In a $3 - psf - graph$ $\widetilde{G}_{P,V} = ((P, \widetilde{\mu}), (P, \widetilde{\rho}))$, every maximal independent set is a minimal dominating set*

Proof. For a maximal independent $\widetilde{S}$ of $\widetilde{G}_{P,V}$. By Theorem 2.4, $\widetilde{S}$ is dominating set in $\widetilde{G}_{P,V}$. If we consider $\widetilde{S}$ being not minimal dominating set in $\widetilde{H}_{P,V}(e_i)$ $\forall$ $e_i \in P$ for $i = 1, 2, \ldots, n$. Then $\exists$ atleast one vertex $a \in \widetilde{S}$ so that $\widetilde{S} \setminus \{a\}$ is a dominating set that means $\widetilde{S} \setminus \{a\}$ dominates $V \setminus (\widetilde{S} \setminus \{a\})$. Thus, there exists atleast one vertex in $\widetilde{S}$ which dominates a. This contradict our assumption. Therefore $\widetilde{S}$ is a minimal dominating set in $\widetilde{H}_{P,V}(e_i)$ $\forall$ $e_i \in P$ for $i = 1, 2, \ldots, n$. Hence $\widetilde{S}$ is a minimal dominating set in $G_{P,V}$. □

3 Conclusions

Due to the large range of applications and domination characteristics that can be defined, domination theory research is interesting. This work introduces the idea of dominating sets, independent sets, domination number etc. for m-polar soft fuzzy graphs and shows some intriguing findings. In a similar situation, future studies can define and examine additional domination parameters.

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