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Different Estimation Methods of the Stress-Strength Reliability Power Distribution

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Abstract

This paper deals with estimation of the reliability system in the stress- strength model of the shape parameter for the power distribution. The proposed approach has been including different estimations methods such as Maximum likelihood method, Shrinkage estimation methods, least square method and Moment method. Comparisons process had been carried out between the various employed estimation methods with using the mean square error criteria via Matlab software package.

Keywords: Power distribution, (S-S) Stress-Strength Reliability, Maximum Likelihood, Moment method, Least Squares method, Shrinkage method.

1. Introduction

The power function distribution represents one of the most important distribution approach in statistical process. It can be expressed by $x \sim pow(x, \alpha)$. Actually, it can be considered as a simple and flexible distribution method that can be used in different applications such as the estimation of reliability for electrical components [1]. This distribution has been introduced in (1967) by Malik H.J throughout studying the exact moment of power function distribution and found a precise expression for moment of power function distribution [2]. Although Bayesian estimation method of parameters have been used by several statisticians and mathematical analysts, but many researchers considered the power function distribution is better than a lot of distributions such as lognormal distribution, exponential distribution and weibull distribution. The power function distribution is as a special case for person first kind distribution that represents the simplicity of the moments for power function distribution [3].
In fact blue estimation method has been introduced for the estimation of the scale and location parameter from the Log-gamma distribution. Many of others researchers were presented estimations of normal distribution parameters by using likelihood function [4-6].

The aim of this work is to estimate the reliability system in the stress- strength model of power function distribution.

This process has been carried out with using and comparing different estimation methods such as Maximum likelihood estimation (MLE), Moment (MOM), least square (LS) and Shrinkage estimation methods (SH). The (c.d.f) of random variable X for power function distribution is given as in the following [7, 8].

$$F(x, \theta, \alpha) = x^{\alpha}\theta^{\alpha} \qquad 0 < x < \theta^{-1} \qquad (1)$$

While the probability density function (P.d.f) of power distribution can be defined as:

$$f(x, \theta, \alpha) = \alpha\theta^{\alpha}x^{\alpha-1} \qquad 0 < x < \theta^{-1} \qquad (2)$$

Where α and θ represent the shape and scale parameters respectively.

In special case, if $\theta = 1$ then (c. d. f) will be as follows:

$$F(x, \alpha) = x^{\alpha} \qquad 0 < x < 1 \qquad (3)$$

And consequently, the (p. d. f) of power function distribution will be:

$$f(x, \alpha) = \alpha x^{\alpha-1} \qquad 0 < x < 1 \qquad (4)$$

The two random variable X and Y are following the power function distribution with parameters $(\alpha_1, 1)$ and $(\alpha_2, 1)$ respectively and according to strength – stress (S-S) model.

Hence, the $x \sim pow(x, \alpha_1)$ and $y \sim pow(y, \alpha_2)$in the strength – stress model can be defined as in the following:

$$R = P(y < x)$$

$$\int_0^1 \int_O^X f(x)\ f(y)\ dy\, dx = \int_0^1 \int_0^x \alpha_1 x^{\alpha_1-1}\alpha_2 y^{\alpha_2-1} dy\, dx$$

Where

$$p(y < x) = R = \frac{\alpha_1}{\alpha_1+\alpha_2} \qquad (5)$$

2. Estimation Methods

2.1 Maximum Likelihood Estimator (MLE)

The maximum likelihood estimator is most popular method because it is approximates the minimum variance unbiased [12]. Let$x_1, x_2 \ldots\ldots x_n$ be a random sample of$pow(\alpha_1, 1)$ and$y_1, y_2 \ldots\ldots y_m$ be a random sample of $pow(\alpha_2, 1)$

Then, the Maximum likelihood forα_1 andα_2 will be:

$$L = L(\alpha_i, x_i, y_i) = \prod_{i=1}^{n} f(x_i) \prod_{i=1}^{m} g(y_i)$$

$$L = \prod_{i=1}^{n} \alpha_1^n\, x_i^{\alpha_1-1} \prod_{i=1}^{m} \alpha_2^m\, y_i^{\alpha_2-1}$$

Taking the logarithm of both sides, then:

$$Ln\, L = n \ln\alpha_1 + (\alpha_1+1)\ln\sum_{i=1}^{n} x_i + m\ln\alpha_2 + (\alpha_2+1)\ln\sum_{i=1}^{m} y_i$$

Taking the partial derivation for the above equations with respect to the shape parameter and then equating the results to zero, this will lead to:

$$\hat{\alpha}_{1_{mle}} = \frac{-n}{\ln\sum_{i=1}^{n} x_i} \qquad (6)$$

$$\hat{\alpha}_{2mle} = \frac{-m}{ln\sum_{i=1}^{m} y_i} \qquad (7)$$

$\hat{\alpha}_{2mle}$in equation (5); the reliability estimation of stress – strength model using maximum likelihood method will become:

$$\hat{R}_{mle} = \frac{\hat{\alpha}_{1_{mle}}}{\hat{\alpha}_{1_{mle}}+\hat{\alpha}_{2_{mle}}} \qquad (8)$$

2.2. Moment Method (MOM)

The moment method can be considered as the most common and a simple method that have been used in estimation of the parameters. It can be summarized by equating the population moments $M_r = E(x^r)$ as in the following [8].

$$E(x) = \frac{\alpha_1}{\alpha_1+1} \text{ and } E(y) = \frac{\alpha_2}{\alpha_2+1}$$

When equating the sample mean *E(x)* and *E(y)* to the corresponding population mean, then

$$\frac{\alpha_1}{\alpha_1+1} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad (9)$$

$$\frac{\alpha_2}{\alpha_2+1} = \frac{\sum_{i=1}^{m} y_i}{m} \qquad (10)$$

From equations (9) and (10), the estimation of parameters α_1 and α_2 using moment method will be written as in the following:

$$\hat{\alpha}_{1_{mom}} = \frac{\sum_{i=1}^{n} xi}{n-\sum_{i=1}^{n} xi} \qquad (11)$$

$$\hat{\alpha}_{2_{mom}} = \frac{\sum_{i=1}^{m} yi}{m-\sum_{i=1}^{m} yi} \qquad (12)$$

Substituting the equations (11) and (12) in equation (5), then the estimation of stress – strength reliability with using moment method will become:

$$\hat{R}_{mom} = \frac{\hat{\alpha}_{1_{mom}}}{\hat{\alpha}_{1_{mom}}+\hat{\alpha}_{2_{mom}}} \qquad (13)$$

2.3 Least Square Method (LSM) [5] [9]

Let $x_1, x_2 \dots\dots x_n$ be a random sample of $pow(\alpha_1, 1)$ and $y_1, y_2 \dots\dots y_m$ be a random sample of $pow(\alpha_2, 1)$, then:

$$F(x_1) = x_i^{\alpha_1}$$

$$x_i = [F(x_i)]^{\frac{1}{\alpha_1}}$$

Taking the logarithm for the both sides of the above equation, then:

$$\ln(x_i) = \frac{1}{\alpha_1}\ln[F(x_i)]$$

$$y = ax + b$$

$$y = \ln x_i, \quad a = \frac{1}{\alpha_i}, \quad x_i = \ln[F(x_i)], \quad b = 0$$

Hence

$$a = \frac{\sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i y_i}{n}}{\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}$$

$$\hat{\alpha}_{1_{Ls}} = \frac{\sum_{i=1}^{n}[\ln F(x_i)]^2 - \frac{\left[\sum_{i=1}^{n}\ln F(x_i)\right]^2}{n}}{\sum_{i=1}^{n}\ln F(x_i)\ln x_i - \frac{\sum_{i=1}^{n}\ln F(x_i)\ln x_i}{n}} \qquad (14)$$

$$\hat{\alpha}_{2_{Ls}} = \frac{\sum_{i=1}^{m}[\ln G(y_i)]^2 - \frac{\left[\sum_{i=1}^{m}\ln G(y_i)\right]^2}{m}}{\sum_{i=1}^{m}\ln G(y_i)\ln y_i - \frac{\sum_{i=1}^{m}\ln G(y_i)\ln y_i}{m}} \qquad (15)$$

For stress – strength reliability $\hat{R}_{Ls}$ using least square method as in the following:

$$\hat{R}_{Ls} = \frac{\hat{\alpha}_{1_{Ls}}}{\hat{\alpha}_{1_{Ls}}+\hat{\alpha}_{2_{Ls}}} \tag{16}$$

2.4 Shrinkage Estimation Method *(sh)* [10][13].

The shrinkage estimation method can be considered as the Bayesian approach which has depended on the prior information. The basic reasons for using the prior estimation had been introduced by Thompson in 1968.In the shrinkage estimation method the parameter was used as initial value α_0 from the past and usual estimator $\hat{\alpha}_{ub}$ through consideration them by shrinkage weight factor, $\emptyset(\hat{\alpha})$, $0 < \emptyset(\hat{\alpha}) < 1$, which can be written as:

$$\hat{\alpha}_{sh} = \emptyset(\hat{\alpha})\hat{\alpha}_{ub} + \big(1-\emptyset(\hat{\alpha})\big)\alpha_0 \tag{17}$$

To find $\hat{\alpha}_{ub}$ from $\hat{\alpha}_{mle}$

$$\hat{\alpha}_{mle} = \frac{-n}{\ln\sum_{i=1}^{n} x_i}$$

$$\frac{n}{n-1}\hat{\alpha}_{mle} \neq \hat{\alpha}_{mle}$$

$$\hat{\alpha}_{ub} = \frac{n-1}{n}\hat{\alpha}_{mle}$$

$$\hat{\alpha}_{ub} = \frac{-(n-1)}{\sum_{i=1}^{n}\ln x_i} \tag{18}$$

2.4.1 Shrinkage Weight Function *(sh_1)* [11].

In this case, the shrinkage weight factor has used as a function of n, in which:

$$\emptyset(\hat{\alpha}) = \left|\frac{\sin n}{n}\right|, \quad \hat{\alpha}_{ub} = \frac{-(n-1)}{\sum_{i=1}^{n}\ln x_i}$$

Substituting in equation (17), then:

$$\hat{\alpha}_{1_{sh_1}} = \left|\frac{\sin n}{n}\right|\hat{\alpha}_{ub} + \left(1-\left|\frac{\sin n}{n}\right|\right)\alpha_0 \tag{19}$$

$$\hat{\alpha}_{2_{sh_1}} = \left|\frac{\sin m}{m}\right|\hat{\alpha}_{ub} + \left(1-\left|\frac{\sin m}{m}\right|\right)\alpha_0 \tag{20}$$

Substituting $\hat{\alpha}_{1_{sh_1}}$,$\hat{\alpha}_{2_{sh_1}}$ in equation (5), then the reliability estimation of stress – strength model with using shrinkage weight function will become:

$$\hat{R}_{sh_1} = \frac{\hat{\alpha}_{1_{sh_1}}}{\hat{\alpha}_{1_{sh_1}}+\hat{\alpha}_{2_{sh_1}}} \tag{21}$$

2.4.2 Constant Shrinkage Factor (sh_2) [11].

In constant shrinkage factor case, it can be assumed that$\emptyset(\hat{\alpha}) = k$, *K*=0.001 which represent the constant shrinkage weight factor. Implying (1-*K*=0.999), then:

$$\hat{\alpha}_{1_{sh_2}} = k_1\hat{\alpha}_{ub} + (1-k_1)\alpha_0 \tag{22}$$

$$\hat{\alpha}_{2_{sh_2}} = k_2\hat{\alpha}_{ub} + (1-k_2)\alpha_0 \tag{23}$$

Substituting$\hat{\alpha}_{1_{sh_2}}$,$\hat{\alpha}_{2_{sh_2}}$ in equation (5), then reliability estimation of stress – strength model with using constant shrinkage factor will become as in the following:

$$\hat{R}_{sh_2} = \frac{\hat{\alpha}_{1_{sh_2}}}{\hat{\alpha}_{1_{sh_2}}+\hat{\alpha}_{2_{sh_2}}} \tag{24}$$

2.4.3 Beta Shrinkage Factor (sh_3) [11].

In this case, the assumption of $\emptyset(\hat{\alpha}) = \beta(1, n, m)$ for the Beta shrinkage weight factor has been taken as $\emptyset(\hat{\alpha}_1) = \beta(1, n)$, and $\emptyset(\hat{\alpha}_2) = \beta(1, m)$ and this implies to the following shrinkage estimators:

$$\hat{\alpha}_{1_{sh_3}} = \beta(1, n)\hat{\alpha}_{ub} + \left(1 - \beta(1, m)\right)\alpha_0 \qquad (25)$$

$$\hat{\alpha}_{2_{sh_3}} = \beta(1, n)\hat{\alpha}_{ub} + \left(1 - \beta(1, m)\right)\alpha_0 \qquad (26)$$

Substituting $\hat{\alpha}_{1_{sh_3}}$, $\hat{\alpha}_{2_{sh_3}}$ in equation (5), then the reliability estimation of stress – strength model with using Beta shrinkage factor will become as in the following:

$$\hat{R}_{sh_3} = \frac{\hat{\alpha}_{1_{sh_3}}}{\hat{\alpha}_{1_{sh_3}} + \hat{\alpha}_{2_{sh_3}}} \qquad (27)$$

3- Simulation Process

The simulation process has done with using different sample size such as (30, 50 and 100) and built on 1000 iteration and using the mean square error (MSE) to measure and check the performance as in the following steps:

Step 1: the random sample generated for X according to the uniform distribution over the interval (0, 1) as $r_1, r_2 \ldots\ldots\ldots\ldots r_n$ and the random sample generated for Y according to the uniform distribution over the interval (0, 1), as $s_1, s_2, \ldots\ldots\ldots\ldots s_m$

Step 2: transforming the above power distribution with using reliability as in the following:

$R(x) = 1 - x^{\alpha}$

$x_i = (1 - r_i)^{\frac{1}{\alpha}}$ i=1, 2, 3 ……..n

Using the same procedure .then

$y_j = (1 - s_j)^{\frac{1}{\alpha}}$ j=1, 2, 3 ……..m

Step 3: calculating $\hat{\alpha}_{1_{mle}}$ and $\hat{\alpha}_{2mle}$ using equations (6) and (7) respectively.

Step 4: calculating $\hat{\alpha}_{1_{mom}}$ and $\hat{\alpha}_{2_{mom}}$ using equations (9) and (10) respectively.

Step 5: calculating $\hat{\alpha}_{1_{Ls}}$ and $\hat{\alpha}_{2_{Ls}}$ using equations (14) and (15) respectively.

Step 6: calculating $\hat{\alpha}_{1_{sh_i}}$ and $\hat{\alpha}_{2_{sh_i}}$ when *i*=1, 2, 3 using equations (19), (20), (22), (23), (25) and (26) respectively.

Step 7: calculating $\hat{R}_{mle}$, $\hat{R}_{mom}$, $\hat{R}_{Ls}$, $\hat{R}_{sh_1}$, $\hat{R}_{sh_2}$ and $\hat{R}_{sh_3}$ using equations (8), (13), (16), (21), (24) and (27) respectively. Where, $\hat{R}$ point to the suggest estimator of reliability *R*. The outcomes results are listed in the **Tables 1, 3, 5, 7, 9, 11, 13, 15, 17.** respectively.

4. Numerical Result

Some methods of goodness of fit analysis are employed here; the measurement give an indication of best method is mean square error (MSE) from tables for all.

1. For all n=(30,50,100) and m=(30,50,100) in this work for minimum mean square error (MSE) for the stress-strength reliability estimator of power function distribution after noted the mean square error in tables, the result indicates that shrinkage estimator (Sh_2) is the best .
2. For all n=(30,50,100) and m=(30,50,100) the minimum mean square error (MSE) for the stress- strength reliability estimator of power function distribution ,we noticed that the shrinkage estimator is the best and follows by maximum likelihood estimator (MLE), moment estimator (MOM) and least square estimator (LS).
3. For the various cases when (n=30 and m=30), (α_1=1and α_2=1) then be moment

estimator (MOM) batter then maximum likelihood estimator (MLE).

Table 1. The estimation when R=0.5, alpha1= 1, alpha2= 1.

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.490139	0.511770	0.500105	0.500003	0.500106	0.491397
	50	0.502377	0.501424	0.485568	0.500000	0.492930	0.498530
	100	0.500411	0.497019	0.485535	0.500002	0.487835	0.495937
50	30	0.497673	0.499256	0.514426	0.500001	0.507071	0.501789
	50	0.501290	0.502633	0.499992	0.499999	0.499969	0.501975
	100	0.501921	0.501610	0.499903	0.499999	0.494837	0.500422
100	30	0.496428	0.499589	0.514526	0.500001	0.512236	0.502114
	50	0.497654	0.498059	0.500099	0.500001	0.505168	0.501268
	100	0.500210	0.500975	0.499999	0.500000	0.499997	0.498823

Table 2. MSE values when R = 0.5, alpha1= 1, alpha2= 1.

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.000519010585	0.000403849563	0.000000057885	0.000000000048	0.000000059360	0.001867492654	mse_{sh2}
	50	0.003335498758	0.00404372155	0.00021100121	0.00000000367	0.00005342323	0.00596515484	mse_{sh2}
	100	0.00275059025	0.00358047587	0.00021203899	0.00000000287	0.00015090743	0.00535912842	mse_{sh2}
50	30	0.00338030723	0.00439697723	0.00021093108	0.00000000358	0.00005343251	0.00621983678	mse_{sh2}
	50	0.00264717587	0.00349108397	0.00000007768	0.00000000277	0.00000120029	0.00505326237	mse_{sh2}
	100	0.00186086110	0.00248158856	0.00000006313	0.00000000196	0.00002730713	0.00387593133	mse_{sh2}
100	30	0.00268746093	0.00362852707	0.00021403837	0.00000000301	0.00015287915	0.00474061679	mse_{sh2}
	50	0.00191579777	0.00247303858	0.00000006418	0.00000000198	0.00002736419	0.00382297898	mse_{sh2}
	100	0.00130008952	0.00169115906	0.00000003502	0.00000000134	0.00000013940	0.00230190166	mse_{sh2}

Table 3. The estimation when R =0.333333, alpha1= 1, alpha2= 2.

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.333767	0.334263	0.333414	0.333336	0.333415	0.336289
	50	0.337382	0.334902	0.320640	0.333333	0.327086	0.336407
	100	0.338877	0.336147	0.320533	0.333332	0.322533	0.335001
50	30	0.336594	0.338421	0.346229	0.333331	0.339564	0.342261
	50	0.333468	0.332847	0.333339	0.333334	0.333357	0.334368
	100	0.335005	0.333733	0.333255	0.333334	0.328789	0.335376

100	30	0.328662	0.329458	0.346444	0.333337	0.344379	0.334704
	50	0.332007	0.332608	0.333424	0.333335	0.337963	0.334676
	100	0.335370	0.335221	0.333327	0.333332	0.333320	0.335713

Table 4. MSE values when R = 0.333333, alpha1= 1, alpha2= 2.

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00333045942	0.00407349011	0.00000456676	0.00000000366	0.00000468648	0.00602348672	mse_{sh2}
	50	0.00272675676	0.00317425319	0.00016326720	0.00000000289	0.00004172719	0.00491135012	mse_{sh2}
	100	0.00235315660	0.00304112894	0.00016601546	0.00000000241	0.00011895110	0.00417668486	mse_{sh2}
50	30	0.00253964500	0.00309479408	0.00016860628	0.00000000272	0.00004151959	0.00486156577	mse_{sh2}
	50	0.0020245050	0.00236857438	0.00000005918	0.00000000211	0.00000091460	0.00367715958	mse_{sh2}
	100	0.00155424555	0.00194082119	0.00000004951	0.00000000158	0.00002113966	0.00293098875	mse_{sh2}
100	30	0.00224635487	0.00265176602	0.00017436372	0.00000000250	0.00012458560	0.00424344399	mse_{sh2}
	50	0.00149852589	0.00180930296	0.00000005236	0.00000000161	0.00002197596	0.00292503967	mse_{sh2}
	100	0.00105247266	0.00125820431	0.00000002811	0.00000000107	0.00000011190	0.00203644507	mse_{sh2}

Table 5. The estimation when R =0.666666, alpha1= 2, alpha2= 1.

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.662728	0.663078	0.666702	0.666668	0.666702	0.664002
	50	0.663332	0.663325	0.653800	0.666670	0.660460	0.658256
	100	0.667976	0.666353	0.653665	0.666666	0.655736	0.664065
50	30	0.662408	0.663825	0.679385	0.666668	0.672938	0.661386
	50	0.666429	0.665317	0.666658	0.666665	0.666634	0.664104
	100	0.666855	0.665989	0.666582	0.666666	0.662053	0.663561
100	30	0.660560	0.663617	0.679517	0.666669	0.677516	0.667085
	50	0.665754	0.666582	0.666742	0.666665	0.671203	0.667305
	100	0.66579	0.665672	0.666668	0.666667	0.666668	0.665879

Table 6. MSE values when R = 0.666666, alpha1= 2, alpha2= 1.

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.251955	0.249262	0.250052	0.250001	0.250053	0.251874
	50	0.253903	0.253116	0.239376	0.250000	0.244759	0.253825
	100	0.253407	0.252539	0.239330	0.250001	0.241002	0.253077
50	30	0.251326	0.252295	0.260959	0.250000	0.255319	0.256074
	50	0.251116	0.249421	0.250005	0.250001	0.250020	0.251890
	100	0.250869	0.250132	0.249939	0.250002	0.246185	0.251854
100	30	0.248037	0.248544	0.261095	0.250002	0.259334	0.252563
	50	0.250922	0.250898	0.250069	0.250000	0.253894	0.254081
	100	0.250477	0.250285	0.250003	0.250000	0.250005	0.250916

Table7. The estimation when R =0.2500000, alpha1= 1, alpha2= 3.

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00328250122	0.00373222061	0.00000425693	0.00000000342	0.00000436837	0.00606980612	mse_{sh2}
	50	0.00259447656	0.00324478044	0.00016767398	0.00000000266	0.00004106095	0.00517968821	mse_{sh2}
	100	0.00206071212	0.00242868274	0.00017126775	0.00000000222	0.00012180771	0.00389842610	mse_{sh2}
50	30	0.00273481415	0.00325502994	0.00016399215	0.00000000291	0.00004210945	0.00532038893	mse_{sh2}
	50	0.00185242541	0.00225935108	0.00000005461	0.00000000194	0.00000084415	0.00353063805	mse_{sh2}
	100	0.00157365989	0.00185528680	0.00000005292	0.00000000166	0.00002184280	0.00302255985	mse_{sh2}
100	30	0.00239094911	0.00285689399	0.00016762461	0.00000000253	0.00012031708	0.00390968491	mse_{sh2}
	50	0.00138303657	0.00166057068	0.00000004423	0.00000000141	0.00002102378	0.00268810353	mse_{sh2}
	100	0.00096451894	0.00119679615	0.00000002584	0.00000000099	0.00000010287	0.00200269936	mse_{sh2}

Table 8. MSE values when R = 0.2500000, alpha1= 1, alpha2= 3.

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00238794868	0.002815755086	0.000003239263	0.000000002596	0.000003324242	0.004110057260	mse_{sh2}
	50	0.00179231243	0.002194623563	0.000114274591	0.000000001810	0.000029189762	0.003461646653	mse_{sh2}
	100	0.00154622400	0.002007092701	0.000115260720	0.000000001531	0.000082459639	0.003073759490	mse_{sh2}
50	30	0.00194448207	0.002350645135	0.000121745483	0.000000002092	0.000030292392	0.003608076782	mse_{sh2}
	50	0.00148586648	0.001741949682	0.00000004300	0.00000000153	0.00000066465	0.00273533263	mse_{sh2}
	100	0.0010774882	0.00133269966	0.00000003359	0.00000000109	0.00001488988	0.00219412488	mse_{sh2}
100	30	0.00152834454	0.001698567219	0.00012497442	0.00000000173	0.00008905011	0.00286844206	mse_{sh2}
	50	0.00112174712	0.001257705394	0.00000003779	0.00000000120	0.00001556871	0.00215713682	mse_{sh2}
	100	0.00072228004	0.000834738472	0.00000001933	0.00000000074	0.00000007694	0.00145595257	mse_{sh2}

Table 9. The estimation when R =0.5000000, alpha1= 2, alpha2= 2.

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.500752	0.500283	0.499984	0.500000	0.499984	0.498106
	50	0.501153	0.500582	0.485629	0.500000	0.492974	0.497582
	100	0.502017	0.500986	0.485552	0.500001	0.487843	0.498328
50	30	0.497610	0.498436	0.514437	0.500001	0.507076	0.501432
	50	0.498133	0.498306	0.500010	0.500002	0.500040	0.495669
	100	0.502684	0.501383	0.499900	0.499999	0.494824	0.500963
100	30	0.497646	0.499452	0.514483	0.499999	0.512190	0.500630
	50	0.499560	0.500307	0.500088	0.499999	0.505127	0.502454
	100	0.500073	0.500067	0.500000	0.500000	0.499999	0.501504

Table 10. MSE values when R = 0.5000000, alpha1= 2, alpha2= 2.

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.004128967671	0.004604492710	0.000005672908	0.000000004557	0.000005821534	0.007466248540	mse_{sh2}
	50	0.003355245833	0.003784728713	0.000209383584	0.000000003620	0.000052863372	0.006185754049	mse_{sh2}
	100	0.002539072228	0.002862493900	0.000211322115	0.000000002679	0.000150482732	0.004649858803	mse_{sh2}
50	30	0.003222287571	0.003616390312	0.0002112752765	0.000000003490	0.000053445804	0.006250665745	mse_{sh2}
	50	0.002543357768	0.002951662489	0.000000077652	0.000000002770	0.000001200648	0.005053937844	mse_{sh2}
	100	0.001813708137	0.001971894924	0.000000061788	0.000000001892	0.000027409095	0.003494129231	mse_{sh2}
100	30	0.002623859568	0.003039695366	0.000212392743	0.000000002745	0.000151354885	0.004670769245	mse_{sh2}
	50	0.001796372291	0.001975020089	0.000000057684	0.000000001826	0.000026875471	0.003170938343	mse_{sh2}
	100	0.001240136539	0.001388572811	0.000000033491	0.000000001284	0.000000133296	0.002447561555	mse_{sh2}

Table 11. The estimation when R =0.750000, alpha1= 3, alpha2= 1.

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.747445	0.747676	0.749984	0.750000	0.749984	0.742832
	50	0.748323	0.748968	0.739063	0.750000	0.744691	0.743426
	100	0.749614	0.749635	0.739003	0.750001	0.740768	0.745187
50	30	0.745730	0.747290	0.760672	0.750000	0.755268	0.746282
	50	0.746656	0.747272	0.750008	0.750001	0.750029	0.742852
	100	0.750657	0.749959	0.749925	0.749999	0.746097	0.748081
100	30	0.746228	0.748209	0.760705	0.750000	0.759030	0.746915
	50	0.748316	0.749575	0.750066	0.749999	0.753825	0.749445
	100	0.748535	0.748006	0.750002	0.750000	0.750005	0.749047

Table 12. MSE values when R = 0.750000, alpha1= 3, alpha2= 1.

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.002384459481	0.002851628898	0.000003192342	0.000000002563	0.000003275999	0.004521397784	mse_{sh2}
	50	0.001948985166	0.002244248987	0.000121340373	0.000000002036	0.000030219158	0.003732969877	mse_{sh2}
	100	0.001431149872	0.001606728294	0.000122472102	0.000000001507	0.000086823942	0.002762765254	mse_{sh2}
50	30	0.001920772013	0.002309071883	0.000115391937	0.000000001963	0.000029585622	0.003739290580	mse_{sh2}
	50	0.001497007264	0.001814282137	0.000000043673	0.000000001558	0.000000675002	0.003051262077	mse_{sh2}
	100	0.001020052304	0.001169978602	0.000000034777	0.000000001064	0.000015586022	0.002017961664	mse_{sh2}
100	30	0.001547120432	0.001996286322	0.000115999905	0.000000001544	0.000083023582	0.002746512587	mse_{sh2}
	50	0.001026872629	0.001229979595	0.000000032430	0.000000001027	0.000014956803	0.001827470288	mse_{sh2}
	100	0.000702148814	0.000841172097	0.000000018293	0.000000000701	0.000000072800	0.001371989596	mse_{sh2}

Table 13. The estimation when R =0.400000, alpha1= 2, alpha2= 3.

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.404506	0.403784	0.399881	0.399997	0.399879	0.403496
	50	0.404397	0.402729	0.386249	0.399998	0.393222	0.399588
	100	0.402856	0.400909	0.386181	0.400001	0.388360	0.398094
50	30	0.398661	0.399451	0.413926	0.400001	0.406817	0.406414
	50	0.403676	0.403770	0.399987	0.399997	0.399947	0.407139
	100	0.401190	0.400025	0.399914	0.400001	0.395066	0.403155
100	30	0.397284	0.397751	0.414066	0.400001	0.411838	0.402213
	50	0.401166	0.401576	0.400081	0.399998	0.404928	0.402906
	100	0.401270	0.401548	0.399996	0.399999	0.399992	0.403123

Table 14. MSE values when R = 0.400000, alpha1= 2, alpha2= 3.

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.003968719837	0.004339244098	0.000005400944	0.000000004337	0.000005542471	0.007343458817	mse_{sh2}
	50	0.002871054638	0.003168491061	0.000191532791	0.000000003008	0.000048857539	0.005249976869	mse_{sh2}
	100	0.002560059256	0.002759431582	0.000193533534	0.000000002729	0.000138181811	0.004430761642	mse_{sh2}
50	30	0.003057266048	0.003368884673	0.000196646030	0.000000003292	0.000049708350	0.005687576189	mse_{sh2}
	50	0.002354827003	0.002572130065	0.000000068179	0.000000002433	0.000001053332	0.004695426152	mse_{sh2}
	100	0.001758583317	0.001851572994	0.000000057177	0.000000001821	0.000024931773	0.003154647116	mse_{sh2}
100	30	0.002509204252	0.002634167448	0.000200568619	0.000000002757	0.000142971140	0.004464657451	mse_{sh2}
	50	0.001776285618	0.001939439022	0.000000056642	0.000000001834	0.000024872335	0.003451614797	mse_{sh2}
	100	0.001103485740	0.001185957806	0.000000029215	0.000000001120	0.000000116266	0.002211734727	mse_{sh2}

Table 15. The estimation when R =0.600000, alpha1= 3, alpha2= 2.

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.598078	0.599079	0.600008	0.600000	0.600008	0.597839
	50	0.599919	0.598806	0.586143	0.600000	0.593246	0.594149
	100	0.602922	0.601586	0.585963	0.599999	0.588188	0.597739
50	30	0.597608	0.597803	0.613749	0.600000	0.606745	0.594849
	50	0.598873	0.599349	0.600002	0.600000	0.600008	0.598037
	100	0.599726	0.598941	0.599914	0.600001	0.595049	0.596674
100	30	0.595986	0.597631	0.613846	0.600000	0.611669	0.598814
	50	0.600409	0.601163	0.600076	0.599998	0.604884	0.602419
	100	0.598798	0.598789	0.600004	0.600001	0.600008	0.599235

Table 16. MSE values when R = 0.600000, alpha1= 3, alpha2= 2.

n	m	msemle	msemom	msesh1	msesh2	msesh3	mseLs	Best
30	30	0.004102721803	0.004522527871	0.000005539858	0.000000004450	0.000005685018	0.006919868113	mse_{sh2}
	50	0.002991823998	0.003292540439	0.000194592389	0.000000003159	0.000048701124	0.005449357116	mse_{sh2}
	100	0.002314174187	0.002431988951	0.000199450713	0.000000002484	0.000142040001	0.004537099349	mse_{sh2}
50	30	0.003193844527	0.003419488337	0.000191806526	0.000000003451	0.000048845005	0.005550811660	mse_{sh2}
	50	0.002287353696	0.002455944918	0.000000066626	0.000000002377	0.000001029306	0.004372047489	mse_{sh2}
	100	0.001805434274	0.001965307620	0.000000059644	0.000000001910	0.000025136702	0.003323110300	mse_{sh2}
100	30	0.002590582075	0.002917038767	0.000194259147	0.000000002723	0.000138841425	0.004619688057	mse_{sh2}
	50	0.001637625023	0.001733549825	0.000000051603	0.000000001680	0.000024385304	0.003120969456	mse_{sh2}
	100	0.001157990638	0.001271271929	0.000000031146	0.000000001194	0.000000123958	0.002418378384	mse_{sh2}

Table 17. The estimation when R =0.500000, alpha1= 3, alpha2= 3.

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.497664	0.497663	0.500090	0.500003	0.500091	0.499114
	50	0.501338	0.501712	0.485595	0.500001	0.492961	0.498385
	100	0.502409	0.501254	0.485551	0.500000	0.487837	0.495447
50	30	0.498054	0.498944	0.514417	0.500000	0.507064	0.501263
	50	0.501699	0.501955	0.499991	0.499998	0.499964	0.501553
	100	0.499867	0.498715	0.499915	0.500001	0.494868	0.498597
100	30	0.496367	0.497546	0.514469	0.500001	0.512188	0.499887
	50	0.500860	0.501257	0.500083	0.499998	0.505135	0.502239
	100	0.499098	0.499134	0.500005	0.500001	0.500010	0.498614

Table 18. MSE values when R = 0.500000, alpha1= 3, alpha2= 3.

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.004060583137	0.004327864388	0.000005599720	0.000000004497	0.000005746462	0.007686836923	mse_{sh2}
	50	0.003215899017	0.003479527896	0.000210304289	0.000000003546	0.000052923876	0.006368191686	mse_{sh2}
	100	0.002691692024	0.002822200371	0.000211674187	0.000000002885	0.000150965205	0.004745640879	mse_{sh2}
50	30	0.003106889720	0.003372070208	0.000210555251	0.000000003353	0.000053185081	0.005623109954	mse_{sh2}
	50	0.002496078235	0.002679722628	0.000000074148	0.000000002646	0.000001145840	0.004620145844	mse_{sh2}
	100	0.001836797867	0.001944809665	0.000000059352	0.000000001906	0.000026943768	0.003330357287	mse_{sh2}
100	30	0.002900030258	0.003015179856	0.000212289915	0.000000003071	0.000151620014	0.004919492350	mse_{sh2}
	50	0.002003087543	0.002106729457	0.000000066242	0.000000002166	0.000027084341	0.003655290108	mse_{sh2}
	100	0.001207869505	0.001273452513	0.000000032502	0.000000001246	0.000000129359	0.002575201627	mse_{sh2}

5. Conclusion

In this absence of real data, we study the performance of the estimator obtained from simulated and the tables, that to estimate the reliability of shrinkage estimator method special constant type shrinkage (Sh2) is the best performance.

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