
A Radar Triangulation System for Guided Missiles

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In any test range firing rockets and guided missiles, a rather complicated instrumentation system is necessary for evaluation of missile performance. An important part of this instrumentation is concerned with the actual trajectory followed by the missile. Flight paths may range from high altitude research rockets reaching altitudes of as much as 100 miles or more to horizontally launched tactical missiles following a low, flat trajectory. Velocities of the order of several thousand miles per hour are obtained, necessitating a system which provides almost instantaneous data. In addition, it was desired that a limited form of control be incorporated in the system, in order that the missiles might be brought down at will.

Space does not permit a comprehensive technical discussion of the entire system. An idea of the complexity of the installation can be gathered from the fact that a total of 662 electron tubes is in use in the basic units required for a simple firing. This figure is exclusive of auxiliary equipment necessary for communications and tactical employment of the system, the associated computer and plotting boards, and similar items normally used in a typical firing. The circuitry of the portion of the system used for measuring propagation time alone accounts for some 328 tubes, including 12 cathode ray tubes, and is capable of measuring time to an accuracy of about 10^{-7} seconds in six simultaneous loops of the system. Geographically, the system is spread over approximately 2,500 square miles of desert and mountain terrain.

In the system, a basic synchronizing signal is obtained from the master timing system and used to initiate a coded double-pulse signal. This double pulse, normally occurring at a rate of 393 pairs per second, is used to key a high-powered transmitter, commonly referred to as the interrogator, which then radiates a double pulse of radio energy in the ultra-high frequency portion of the spectrum. This interrogation signal covers the entire test range, in order that the beacon installed in the missile may receive it at all points of the trajectory. (This double pulse system is used to obtain a high degree of noise and interference immunity for the beacon, which is actuated only when a double-pulse of the proper spacing is received.) Reception of this signal by the beacon causes it, in turn, to send out a reply pulse on a second frequency. This reply signal is then received by all ground stations involved in the system, including the master station at which the interrogation originated. Four remote repeater stations located around the periphery of the range also receive this reply signal and send a relayed reply signal back to the master station on their individual frequencies.

By now most people are familiar with the basic principle of radar, which underlies the theory of the system. In brief, distance can be determined by

measuring to a high degree of accuracy the time required for a pulse of radio frequency energy to travel out to a point in space and return to the point of origin. Since the velocity of propagation is an accurately known constant, the time required is proportional to the distance, and a suitable increment of time can be chosen to represent distance directly. This has been done in the system under discussion.

The master synchronizing signal which initiates the interrogation signal from the master station is obtained from a crystal controlled oscillator by successively counting down in frequency. The crystal frequency has been so chosen as to provide a period equal to the time required for a radiated signal to travel 10,000 feet. Division and multiplication of this basic frequency by factors of ten provide additional periods which represent 100,000 and 1,000 foot intervals. The master synchronizing signal initiates the timing sequence, and the ranges to be measured are displayed along with range marks obtained from the crystal controlled time periods. A separate channel of the indicating portion of the system is used for the direct beacon reply and each of the remote station relayed reply signals. Each channel involves a triplicate oscilloscopic display on which the total distance traversed by the signal in the channel is indicated. After suitable gating and shaping, a pulse derived from each reply is fed into the appropriate channel range indicator to provide a visual indication of the distance involved. The triple display of each channel provides cathode ray tubes on which successively smaller increments of distance may be read. The first trace gives coarse readings to 100 kilofeet intervals, the second trace expands the last 100 kilofeet and gives a reading to 10 kilofeet, and the final trace expands the last 10 kilofeet, with marker intervals of one kilofeet. Interpolation between markers then gives the final reading in 100 foot increments. The bank of cathode ray tubes on which the above indications are presented is then photographed with a 35 mm. camera to obtain a permanent record. The data on this film record can then be read out and reduced by simple mathematical calculations at leisure.

Let the measured path lengths for the main station beacon reply and two remote station replies be represented by

D_1 = path length from main station to beacon and back

D_2 = path length from main station to beacon to first remote station and back

D_3 = path length from main station to beacon to second remote station and back.

Choosing a co-ordinate system such that all three ground stations lie in the X,Y plane, with the origin at the main station and the Y axis passing through the first remote station, the station co-ordinates may be represented by

Main station: 0, 0, 0

First remote station: 0, K, 0

Second remote station: L, M, 0

The distances from the remote stations to the main station are then seen to be respectively

First remote station: K

Second remote station: $N = \sqrt{L^2 + M^2}$

The distances from each station to the missile can now be determined from the measured path lengths:

$$\text{Main station: } A = \frac{D_1}{2}$$

First remote station: $B = D_1 - A - K$

Second remote station: $C = D_2 - A - N$

Letting the co-ordinates of the missile be represented by x , y and z , three simultaneous equations can be obtained:

$$(1) A^2 = x^2 + y^2 + z^2$$

$$(2) B^2 = x^2 + (y - K)^2 + z^2$$

$$(3) C^2 = (x - L)^2 + (y - M)^2 + z^2$$

Subtracting (2) from (1):

$$A^2 - B^2 = 2Ky - K^2$$

$$A^2 - B^2 + K^2$$

$$(4) y = \frac{A^2 - B^2 + K^2}{2K}$$

Since A and B are known from the measured distances and K is a known constant, the value of y may now be calculated.

Subtracting (3) from (1)

$$A^2 - C^2 = 2Ly - L^2 + 2Mx - M^2$$

$$A^2 - C^2 + L^2 + M^2 - 2Ly$$

$$(5) x = \frac{A^2 - C^2 + L^2 + M^2 - 2Ly}{2M}$$

Inserting the known values for A , C , L , and M and the value of y determined from equation (4) allows calculation of the value of x .

Rearranging equation (1):

$$z^2 = A^2 - x^2 - y^2$$

$$(6) z = \sqrt{A^2 - x^2 - y^2}$$

Substitution of the calculated values of x and y allows solving for z .

Trajectory data are normally presented with the launching point as the origin in a level orthogonal system with the Y axis as true north. The data obtained in the above calculations are easily converted to the desired form by making three rotations to obtain the desired orientation of the axes, then translating the origin to the launching point.

In addition to the primary function of triangulation, two other functions are incorporated in the system. A "command" function has been provided whereby a change in the basic pulse repetition rate of the interrogation signal from 393 and 854 pulse pairs per second actuates a relay in the beacon. This is the rather crude remote control system mentioned previously. A second function known as "fail-safe" has a somewhat similar effect. If no interrogation is received by the beacon for a period of about two seconds, a relay is also actuated. Normally, this second function is used to destroy the missile in the event that a failure occurs in the system and thus the whereabouts of the missile cannot be determined. The "command" function is normally used to separate the instrumentation compartment of a research missile from the main body, thus allowing parachute descent of the instrumentation proper.

Associated with the system is an analog computer. This unit converts the pulse-time data obtained with the triangulation system to DC voltages proportional to the distances for each loop. The necessary calculations are then performed electronically, resulting in automatic instantaneous data reduction. The resultant values of the x , y , and z co-ordinates are then plotted automatically on two servo-driven plotting boards. One board presents all three co-ordinates as a function of time, while the other plots x and z co-ordinates versus the y co-ordinate. The accuracy obtainable here is, of

course, considerably less than that attainable by manual reduction of the data, but the convenience of an instantaneous automatic trajectory plot greatly outweighs the consideration of accuracy for many purposes, particularly when it is desired to locate the point of impact for the missile as soon as possible by a ground recovery party.

A second triangulation system in use in New Mexico has also been operated by Research Foundation personnel for the past five years. This system is fundamentally similar to the system just described and was, in fact, the predecessor of the new system. The basic difference between the two systems is the substitution of a radio synchronizing link for the relayed reply link. The data obtained are thus distance differences at each station rather than distance sums at one central station. A disadvantage is the necessity of separate data recording at each station and subsequent time correlation between the individual records so obtained. In essence, the second system performs the time correlation automatically by the single photographic record at the main station.

The second system also incorporates an additional provision for telemetering information from the missile to the ground stations. A pulse-time modulation system is used, using the normal beacon triangulation "reply" pulse as a reference pulse. Two delayed telemetering pulses then follow this reference pulse, with the delay time proportional to a modulating DC voltage for each pulse. Each pulse may be shifted a total of 150 microseconds with respect to its reference zero position, and is used to indicate standard data deviations of from 0 to 5 volts DC. Transducers are used to obtain this standard voltage range from whatever data it is desired to telemeter. The resultant train of three pulses is received at each ground station and measurements of relative pulse displacement between the reference pulse and the two data pulses can then be made to recover the original data.

As a still further refinement of this telemetering system, a digital electronic recorder is used at the main station. This allows automatic reduction of the telemetered data. In this unit two pulsed oscillators are started by the reference pulse, and each is stopped later by the associated telemetering pulse for one channel. A binary counter chain is then used to totalize the number of cycles of oscillation so obtained in each channel. The frequency is so chosen as to make each cycle represent a datum increment of 0.025 volts. The total count obtained is displayed on a bank of neon indicators, which are photographed to obtain a permanent record. The film record is then fed through a second unit at low speed. Here a light beam is projected through the film to a bank of photoelectric cells. Each photocell corresponds to the position of one of the neon indicators and thus is activated whenever a clear spot on the film indicates that the associated neon count indicator was fired. The photocells then actuate a set of relays so connected as to control a pair of electric typewriters. The typewriters then type up, automatically, a complete permanent record of the data obtained throughout the period of missile flight.
