On the Question of the Analysis of $J/\psi \to \rho\pi \to \pi^+\pi^-\pi^0$

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Abstract

This paper presents a method for the analysis of process $J/\psi \to \rho\pi \to \pi^+\pi^-\pi^0$ based on the consideration of the angles of expansion of finite pion pairs. The proposed approach makes it possible to effectively carry out a selection of events in both neutral and charge-conjugate modes of the decay of $J/\psi \to \rho\pi$. The application of the method for the analysis of similar three-body decays in some cases will simplify the analysis and refine current results.

Keywords: charmonium, J/ψ, $\rho\pi$
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1. INTRODUCTION

The interest in the issue of effective separation of the $J/\psi \to \rho\pi$ decay modes arose from considering the experimental results of the analysis of the J/ψ decay into three pions. The main contribution to this process is determined by the decay of $J/\psi \to \rho\pi$ followed by the decay ρ meson into two pions. The probability of this process was measured in a number of experiments [1, 2, 3, 4, 5, 6, 7, 8, 9] and is the highest among J/ψ decays with an intermediate hadron resonance $\mathcal{B}_{\rho\pi}(J/\psi) = 1.69 \pm 0.15\%$ [10]. In addition, the ratio of partial widths $\Gamma_{\rho^0\pi^0}/\Gamma_{\rho\pi}$ was measured in [1, 2, 3, 4, 6], where taking into account the result of the latest experiment [6], the PDG [10] gives the value $\mathcal{B}_{\rho^0\pi^0}(J/\psi) = (5.6 \pm 0.7) \times 10^{-3}$. A feature of the available experimental measurements of the $\mathcal{B}_{\rho\pi}(J/\psi)$ value is a significant discrepancy between the results of early experiments [1, 2, 3, 4, 5, 6, 7] and later measurements by the collaborations BES [8] and BABAR [9]. The resulting error scaling factor, according to PDG [10], is 2.4. These reasons make it interesting to continue the study of both the $J/\psi \to \rho\pi$ process and the entire set of J/ψ decay processes leading to a three-pion final state.

The distribution of squared invariant masses of pion pairs $\pi^+\pi^0$, $\pi^-\pi^0$, and $\pi^+\pi^-$ is shown in three dimensions in Figure 1. Any of the three 2D projections of a 3D distribution is a Dalitz plot [11]. Plots of this kind are widely used in the analysis of various processes, including those for the three-pion J/ψ meson decay. The complexity of measuring the partial probabilities of modes $J/\psi \to \rho^+\pi^-$, $J/\psi \to \rho^-\pi^+$, and $J/\psi \to \rho^0\pi^0$ in such an approach is associated with a significant overlap of distributions of squared invariant masses of pions pairs, as well as the need for a reliable description of the resolution function for a two-dimensional distribution over invariant masses. The inaccuracies of this function, as a rule, introduce a noticeable systematic uncertainty into the result.

2. THE MAIN IDEA OF THE ANALYSIS OF THE PROCESS $J/\psi \to \rho\pi$

As an alternative to the two-dimensional fitting of Dalitz plots, one can propose the following analysis procedure. At the initial stage, three subsets of events are selected in accordance with the following conditions: $\cos\theta_{\pi^+\pi^0} > \cos\theta_{\pi^+\pi^-} \wedge \cos\theta_{\pi^+\pi^0} > \cos\theta_{\pi^-\pi^0}$, $\cos\theta_{\pi^-\pi^0} > \cos\theta_{\pi^+\pi^-} \wedge \cos\theta_{\pi^-\pi^0} > \cos\theta_{\pi^+\pi^0}$, and $\cos\theta_{\pi^+\pi^-} > \cos\theta_{\pi^-\pi^0} \wedge \cos\theta_{\pi^+\pi^-} > \cos\theta_{\pi^+\pi^0}$. Here and further, $\theta_{\pi^+\pi^0}$, $\theta_{\pi^+\pi^-}$, and $\theta_{\pi^-\pi^+}$ are the angles between the momentum vectors of the corresponding π mesons.

Figure 2 shows the distribution of events over the values of the cosines of the angles of expansion of pion pairs in a three-dimensional form. The criteria listed above determine the boundaries of the regions of three subsets of events, where one or another mode of the process under study predominates.

The next stage of the analysis consists in constructing the distributions of the invariant mass of the positively and negatively charged ρ meson according to the first and the second subsets of events, as well as the distribution of the invariant mass ρ^0 meson over the events of the third subset. The simultaneous fitting of the three resulting distributions involves the calculation of all necessary parameters, as in the case of fitting two-dimensional Dalitz plot. Let us discuss the advantages of the proposed approach.

Consider the events corresponding to the selection conditions $\cos\theta_{\pi^+\pi^-} < \cos\theta_{\pi^-\pi^0} \vee \cos\theta_{\pi^+\pi^-} < \cos\theta_{\pi^+\pi^0}$. These criteria allow rejecting most of the $J/\psi \to \rho^0\pi^0$ events, which makes it possible to single out the $J/\psi \to \rho^+\pi^-$ and $J/\psi \to \rho^-\pi^+$ "conditional" in its purest form.

Figure 3 shows the distribution of the invariant mass for the $\pi^-\pi^0$ pair for events that meet the above criteria. The resulting histogram is marked with a dotted line. The area with horizontal blue hatching meets the condition $\cos\theta_{\pi^-\pi^0} > \cos\theta_{\pi^+\pi^0}$, and the part shaded with vertical red lines is the inverse relation $\cos\theta_{\pi^-\pi^0} < \cos\theta_{\pi^+\pi^0}$. The area of the intersection of the distributions determined by this condition is approximately 7.4% of all events in the histogram.

Consider now the distribution of the cosine of the angle between the momentum vectors of π^- and π^0 under the same conditions in Figure 4. The overlap area of the event subset for one-dimensional distributions in magnitude $\cos\theta_{\pi^-\pi^0}$ in this case is 2.3%. The resulting value characterizes the overlap of subsets of events $J/\psi \to \rho^+\pi^-$ and $J/\psi \to \rho^-\pi^+$ on the three-dimensional plot of the angles of expansion of π meson pairs.

A similar parameter at the boundaries of the intersection of charged modes with a subset of events neutral mode is around 2.5%. In all possible variants of intersections, approximately a threefold advantage remains compared to the values of overlapping distributions of invariant masses.

The event compaction effect of the "corners" area in Figure 2 is a consequence of a simple fact. The shell or boundary of

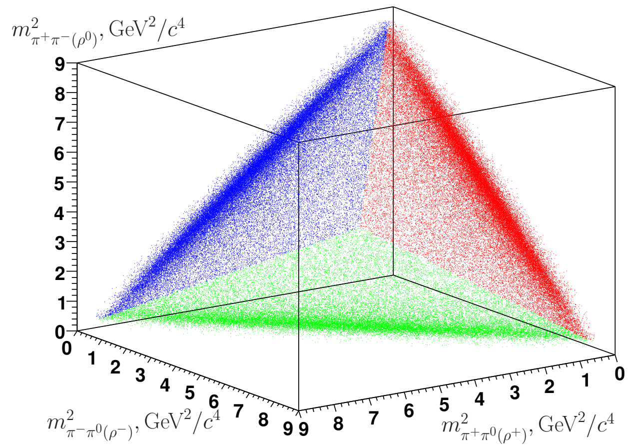

FIGURE 1: The distribution of events over the squares of the invariant masses of pion pairs $m^2_{\pi^+\pi^0}$, $m^2_{\pi^-\pi^0}$, $m^2_{\pi^+\pi^-}$ obtained by MC simulation of the process $J/\psi \to \rho\pi$. The simulation was performed within the framework of the KEDR experiment software [12]. The red, blue, and green regions correspond to the conditions of selecting the J/ψ decay modes $\rho^+\pi^-$, $\rho^-\pi^+$, and $\rho^0\pi^0$, respectively, as described in the text of the article.

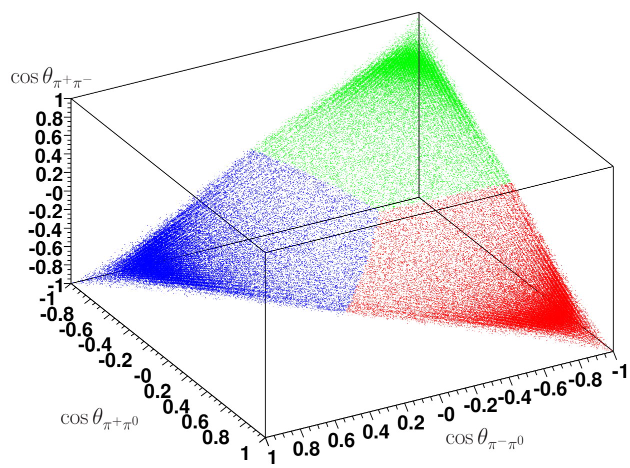

FIGURE 2: Simulation data of the process $J/\psi \to \rho\pi$. The distribution of events over the cosines of the angles $\cos\theta_{\pi^+\pi^0}$, $\cos\theta_{\pi^-\pi^0}$, and $\cos\theta_{\pi^+\pi^-}$ is given. The red, blue, and green regions correspond to the conditions that single out, respectively, the J/ψ meson decay modes $\rho^+\pi^-$, $\rho^-\pi^+$, and $\rho^0\pi^0$.

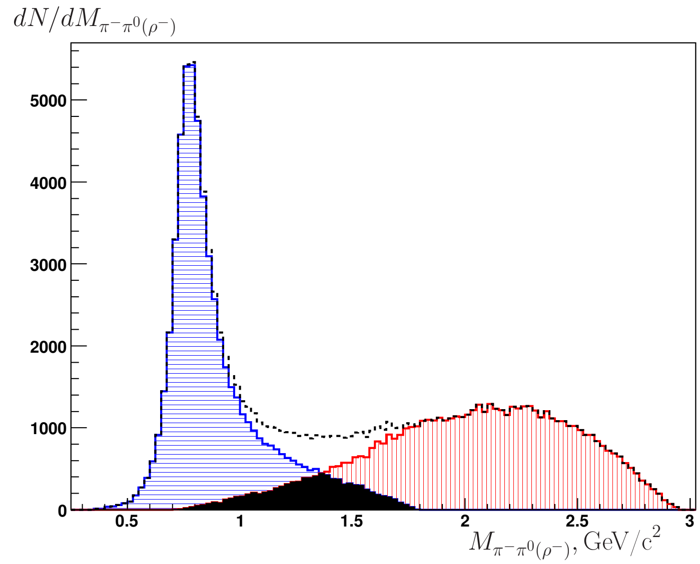

FIGURE 3: The invariant mass distribution of the pair π^- and π^0 under the conditions $\cos\theta_{\pi^+\pi^-} < \cos\theta_{\pi^-\pi^0} \vee \cos\theta_{\pi^+\pi^-} < \cos\theta_{\pi^+\pi^0}$.

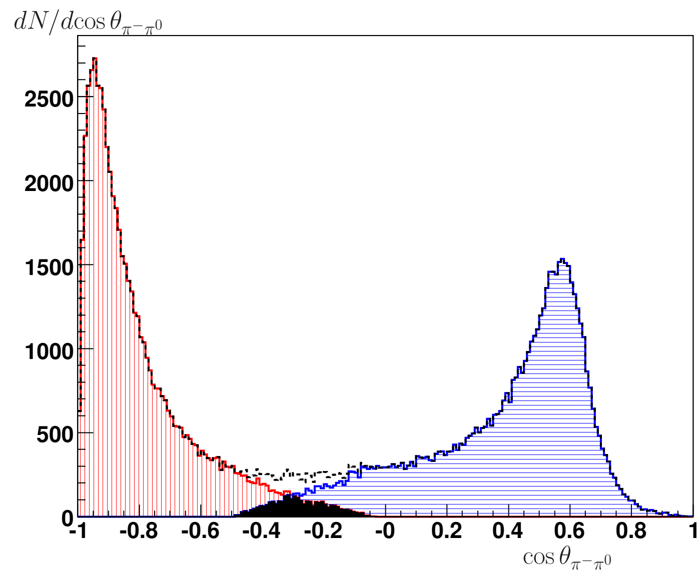

FIGURE 4: Distributions of the cosine of the angle between π^- and π^0 under the conditions $\cos\theta_{\pi^+\pi^-} < \cos\theta_{\pi^-\pi^0} \vee \cos\theta_{\pi^+\pi^-} < \cos\theta_{\pi^+\pi^0}$.

the area occupied by the Dalitz plot for a three-pion decay corresponds to "collinear" events, when the momenta of two particles are directed against the direction of motion of the third particle. In this case, the events of each of the $J/\psi \to \rho\pi$ decay modes are located in the region close to the corresponding side of the triangle in Figure 1, and on the cosine plot of the expansion angles of π meson pairs in the corresponding part of the 3D drawing in Figure 2.

It is possible to find a relation between the observed angle of expansion of a pair of charged pions and their invariant mass, in an approximation where pion masses can be neglected. In this case, $2E_1E_2(1-\cos\theta_{12}) = m^2_{12}$, where E_1 and E_2 are energies of the corresponding pions. Consider the $\pi^-\pi^0$ pair. Under the condition $\cos\theta_{\pi^-\pi^0} > \cos\theta_{\pi^+\pi^-} \wedge \cos\theta_{\pi^-\pi^0} > \cos\theta_{\pi^+\pi^0}$, the observable value is $\cos\theta_{\pi^-\pi^0} > -0.5$ (Figure 4). From here, one can determine that $m^2_{12} < 3E_1E_2$. The maximum of the product E_1E_2 is reached at $E_1 = E_2 = \frac{s-m^2_\pi+m^2_{12}}{4\sqrt{s}}$; hence $m^2_{12} < \frac{3s}{10}$. Holding the next order of smallness in pion mass leads to $m^2_{12} < \frac{8}{5}(\frac{3s}{16} + 4m^2_\pi)$. In numerical form, this corresponds to the condition $m_{12} < 1.73\ \mathrm{GeV}/c^2$ (for $s = M^2_{J/\psi}$).

Using the above conditions on the cosines of the pion pair expansion angles, it is possible to represent the entire set of events in the form of one-dimensional distributions of invariant masses for pion pairs, which is demonstrated in Figures 5, 6, and 7. The upper bound of each of the distributions agrees well with the constraint found above.

Consider the problem of optimizing the mode factorization procedure for the process $J/\psi \to \rho\pi$; that is, we determine the possible region of each of the modes so that the sum of the

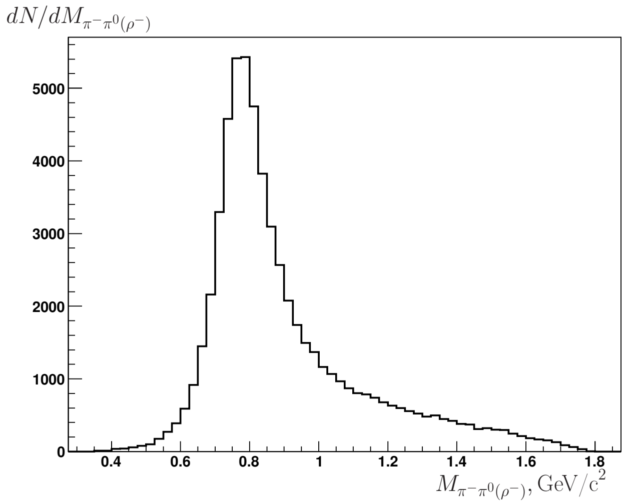

FIGURE 5: Distribution of the invariant mass of the pair $\pi^-\pi^0$ under the conditions $\cos\theta_{\pi^-\pi^0} > \cos\theta_{\pi^+\pi^-} \wedge \cos\theta_{\pi^-\pi^0} > \cos\theta_{\pi^+\pi^0}$.

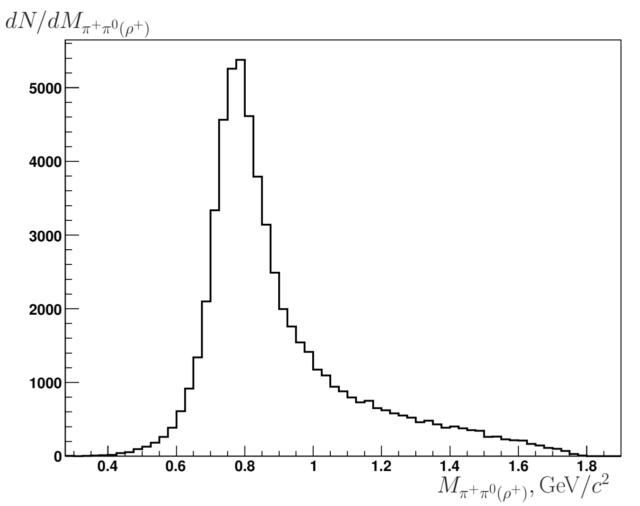

FIGURE 6: Distribution of the invariant mass of the pair $\pi^+\pi^0$ under the conditions $\cos\theta_{\pi^+\pi^0} > \cos\theta_{\pi^+\pi^-} \wedge \cos\theta_{\pi^+\pi^0} > \cos\theta_{\pi^-\pi^0}$.

overlaps of the subsets ε is minimal. Introduce the parameter δ and change "$\cos\theta_{\pi^+\pi^-}$" to "$\cos\theta_{\pi^+\pi^-} + \delta$" in all conditions listed above. Positive values of δ correspond to an increase in the region where the events of the $J/\psi \to \rho^0\pi^0$ mode are distinguished, and negative values correspond to its decrease. Figure 8 shows the resulting dependence.

The optimal value of δ is found to be equal to -0.3. At the same time, the region of the minimum of the chosen optimization function is rather wide, and its difference from the value at zero δ is insignificant. Modification of the criteria on $\cos\theta_{\pi^+\pi^0}$ and $\cos\theta_{\pi^-\pi^0}$ by introducing additional offsets is not required due to the symmetry selection conditions for these parameters, which is confirmed by numerical calculation.

The observed asymmetry of the optimal boundaries of the selected events is associated with a lower efficiency of registration of the $J/\psi \to \rho^0\pi^0$ process compared to charged modes. When analyzing real experimental data, the optimal criteria for mode factorization may not correspond exactly to the conditions described in this article due to the features of the detector, the influence of background events, and the interference of the decay under study with other processes. Nevertheless, selection criteria for the angles of expansion of pion pairs of registered π mesons in the process $J/\psi \to \rho\pi$ significantly increase the efficiency of separation of various modes of a given decay.

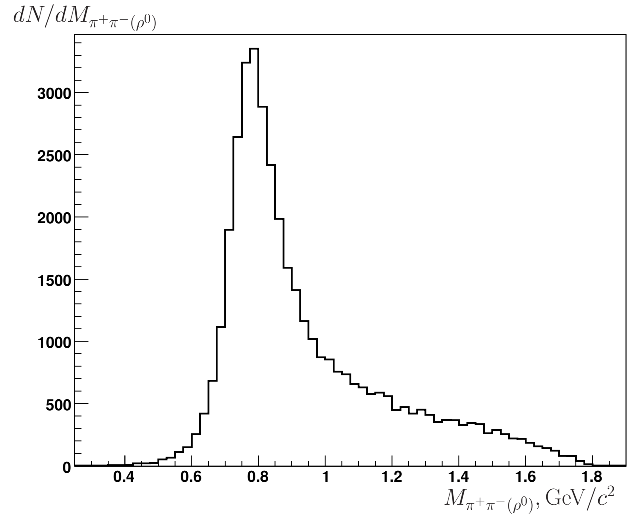

FIGURE 7: Distribution of the invariant mass of the pair $\pi^+\pi^-$ under the conditions $\cos\theta_{\pi^+\pi^-} > \cos\theta_{\pi^-\pi^0} \wedge \cos\theta_{\pi^+\pi^-} > \cos\theta_{\pi^+\pi^0}$.

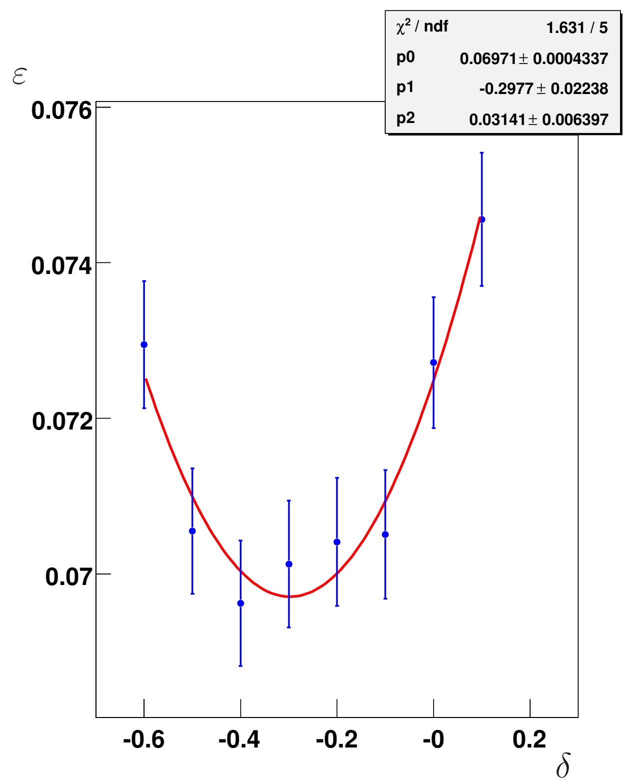

FIGURE 8: Dependence of the sum of overlaps of subsets of distinguished events ε on the value δ, which sets the shift to the value $\cos\theta_{\pi^+\pi^-}$ under the conditions that determine the boundaries of the regions of the J/ψ decay modes under study.

Such criteria are more efficient than restrictions on the squares of the invariant masses and the invariant masses themselves for processes of this kind, if only one-dimensional distributions are considered. Factorization of different $J/\psi \to \rho\pi$ decay modes can be useful for a more accurate measurement of the partial probability of this process.

3. CONCLUSION

A number of critical remarks should be made. The problem of interference of the main $\rho\pi$ channel with the $J/\psi \to \rho(1450)\pi$ decay and other possible processes leading to the three-pion state is not considered here, but the corresponding analysis of one-dimensional distributions of the constructed invariant masses preserves information about interference. Such a one-dimensional analysis is simpler than the description of the elements of a two-dimensional distribution on the Dalitz plot, since a reliable description of the resolution function in a multi-dimensional space is quite a difficult task. The two-dimensional approach in describing the distribution of events over invariant masses on the Dalitz plot carries more information, but the proposed alternative is more efficient to measure the partial probability of the process $J/\psi \to \rho\pi$. This is due to the fact that the main part of the events of this process is concentrated in the "corners" of the three-dimensional distribution of the cosines of the expansion angles of π meson pairs. Regions where one or another mode of the $J/\psi \to \rho\pi$ process predominates can also be distinguished by other conditions that include the main set of events of each considered mode. However, this does not affect the nature of the distributions over the invariant mass for pairs of pions, which will be characterized by a "tail" falling to the right. This, ultimately, provides the advantage of such an analysis in finding the systematic uncertainties associated with the interference with resonances lying above the ρ meson.

The construction of Dalitz plots and selection criteria based on the restriction of the squares of the invariant masses, or the invariant masses themselves, are familiar tools of modern high-energy physics used to analyze the processes leading to three particles in the final state. At the same time, for the analysis of such processes, plots and selection criteria based on the values of the cosines of the expansion angles between the final particles can be considered. The application of the method described above for determining the probabilities of various $J/\psi \to \rho\pi$ decay modes will allow us to check more accurately the conservation of isotopic invariance for the process under consideration. The proposed method can be useful in the study of the charge asymmetry in the decays $J/\psi \to \rho\pi$ and the so-called $\rho - \pi$ puzzle [5], if we carry out a similar analysis of various $\psi(2S) \to \rho\pi$ decay modes. Also, the described method of analysis can be used in the study of the decays $\Upsilon(1S), \Upsilon(2S) \to \pi^+\pi^-\pi^0, \phi K^+K^-, \omega\pi^+\pi^-, K^{*0}(892)K^-\pi^+$ and other processes with similar kinematic features.

CONFLICTS OF INTEREST

The author declares that there are no conflicts of interest regarding the publication of this paper.

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