Managerial Decisions and the Weighted Average Cost of Capital

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Abstract

This paper explores the relationship between the weighted average cost of capital (WACC) of a company and its marginal cost of capital. We adopt the classic economic theory of production to shed light into the conditions upon which WACC can be safely treated as the marginal cost of a company. Any violations of these conditions will lead a company to reach suboptimal capital budgeting decisions.

I. Introduction

This paper examines an important concept central to corporate finance — the weighted average cost of capital ("WACC"). Students and practitioners alike have long been told that to maximize a company's long-term market value, the company needs to maximize the number of positive net present value ("NPV") projects it undertakes. To calculate the NPV of a project, one would simply discount the expected future net cash inflows by the company's WACC. Alternatively, one should accept as many projects as possible so long as the project's internal rate of return (IRR) is greater than the company's WACC.

This IRR rule in capital budgeting, by equating the IRR of a project with the company's WACC, on the surface treats the WACC as the company's marginal cost of capital. As a result, it is easy to mistakenly believe that the WACC is the same as the marginal cost of capital of the company. In fact, a critical assumption of this "capital budgeting rule" is that the company has maintained the target capital structure whereby at that optimal level of debt, the WACC is minimized. However, the road to this WACC minimization is rarely discussed in detail. This paper adopts the classic economic theory of production to shed light into the conditions upon which WACC can be safely treated as the marginal cost. Any violations of these conditions will lead the company to reach suboptimal capital budgeting decisions.

This paper is organized as follows: in the next section, we discuss Modigliani and Miller's (MM) Proposition III since MM provide the fundamental treaties on the estimation of a company's WACC. Section 3 examines the typical textbook treatment of WACC and its relationship with MM's definition. The pedagogical issues are discussed in Section 4. Section 5 concludes the paper.

III. Modigliani and Miller's WACC

In their seminal paper (Modigliani and Miller 1958), under several restrictive assumptions, MM show that the net worth of current shareholders will increase if the expected rate of return on investment is greater than the firm's cost of capital (Proposition III). MM define the firm's cost of capital as:

$$WACC = K^U_s (1 - t\, \Delta B / \Delta I); \qquad (1)$$

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where K^U_s is the cost of equity for the unlevered firm; t is the tax rate; ΔB is the change in debt and ΔI is the new investment. In a later paper (Modigliani and Miller 1963), MM argue that at least as a first approximation, $\Delta B / \Delta I = B^* / (B^* + S^*)$ where $B^* / (B^* + S^*)$ denotes the long-run "target" debt ratio which is constant. However, there were no formal derivations of what the "target" debt ratio should be.

III. Textbook WACC

The usual expression of WACC in textbooks (see, e.g. Brigham and Ehrhardt 2005, and Ross, Westerfield and Jordan 2006), assuming that the firm does not issue preferred stocks, is as follows:

$$WACC = K_d (1 - t) \times B/(B + S) + K_s \times S/(B + S); \qquad (2)$$

where WACC is the company's overall financing cost, K_d is the company's before tax cost of debt, t is the marginal tax rate; K_s is the cost of equity; B is the market value of debt and S is the market value of equity.

This textbook WACC can be shown as identical to MM's definition of WACC by substituting MM's Proposition II into eq. (2) above and assuming that $B / S = \Delta B / \Delta S$ (see Copeland, Weston and Shastri 2005, p. 570 for proof.) [3]

IV. Managerial Issues

Most available textbooks often ignore the fact that the WACC is the <u>average cost of capital</u>, not the marginal cost of capital. However, when a company applies its WACC to discount future cash flows and to decide whether to accept or reject a particular project, the company is treating the WACC as the marginal cost. Basic economic theories show that to maximize a company's value, a company should continue undertaking additional projects until the point where the marginal revenue of an additional project is equal to the marginal cost of the project. (See, e.g., Baumol and Blinder 2000, p. 176). Broadly speaking, if we consider the continuous function and the new project being infinitely small, this condition is the same as investing until the point where the internal rate of return of the new unit (however the unit is defined) equals the capital cost (in percentage) of investing in the new unit. In corporate finance, we can then speak of investing until the point where the marginal return (MR) on the next project equal the marginal cost of the next project, MR = MC; or MR = WACC if we assume that the risk of the new project is similar to that of the company's average project and the company's WACC equals its MC.

In fact, we are using an average cost as if it were a marginal cost. But economic principles indicate that such an application of average cost will not maximize overall company value. So why are we using it this way? It should be noted that following MM's argument, if B/(B + S) is treated as the long run target debt to value ratio (which is constant), then the WACC

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3 MM's Proposition II states that $K^L_s = K^U_s + (K^U_s - K_d)(1 - t)(\Delta B / \Delta S)$, where K^L_s is the cost of equity of a levered firm. Note that eq. (1) can be alternatively expressed as $WACC = K^U_s (1 - t B / B + S)$ following MM's assumption of having the long-run "target" debt ratio.

will be constant since both K_d and K_s are constant in the MM world with perfect competition and identical risk class. As long as the WACC is constant, it is identical to the firm's marginal cost. But this nuance is often ignored. In any case, the insertion of the long term target debt to value ratio argument merely solves the issue of treating the WACC as a marginal cost *mathematically*; there is very little economic rationale behind the argument. This paper shows an alternative demonstration that the WACC can in fact be the marginal cost under certain conditions.

Microeconomic principles suggest that in the long run, all costs are variable. At the point where the long-run average total cost of production is minimized, the company's short-run average total cost of production is identical to the long-run average total cost of production; so is its marginal cost of production. That is, at that particular long-run level of output:

LRATC=SRATC=MC;

where LRATC is the long run average total cost; SRATC is the short run average total cost; and MC is the marginal cost. This argument is discussed in all microeconomic books and should be familiar to students. Figure 1 illustrates the condition. In the short run, the company targets it production level at a certain range and therefore, the minimum cost output level is where SRATC = MC. However, in the long run, the company is faced with many choices of output levels, the SRATC will shift and one can trace the LRATC that "envelopes" the SRATCs at different output levels.[4]

A. The Short-Run Solution

The company's capital costs and investments can be viewed in the same light. Many researchers portray the WACC of the company as a step function as shown in Figure 2A. When the dollar amount of new investments is low and the need for new capital is low, the WACC is low. As the dollar amount of new investments increases, the WACC will inevitably move up in steps. The reasons why the WACC rises in steps rather than continuously is that the financial market is not frictionless. For example, within certain reasonable range of debt level, the perceived risk level of the company will likely remain the same. As a result, K_d and K_s will be relatively constant at that range. By superimposing the investment opportunity schedule (IOS)[5] onto the WACC schedule, one can determine the optimal level of investments in the short-run. As long as the company's WACC is the same as its MCC, the solution is correct and the level of investments will maximize the company's value.

Obviously, the WACC schedule in Figure 2A ignores the flotation costs.[6] In fact, the calculation of WACC as expressed in eq. (2) underestimates the true WACC by focusing only on K_d and K_s before flotation costs. If K_d and K_s are adjusted for flotation costs, then this adjusted WACC curve is similar to the usual short-run average total cost (per unit of production) curve found in most economic textbooks. (See McConnell and Brue, 2002, Chapter 22.) Now we can

[4] The LRATC curve is the points of tangency of all SRATCs.
[5] IOS is a graph ranking the IRRs of each prospective project.
[6] If the WACC curve is indeed the MCC curve, then flotation costs should not matter in the short-run since flotation costs can be considered fixed costs within a certain level of capital raised. MC is related only to variable costs, not to fixed costs.

envision the adjusted WACC curve (or the average total cost per unit of capital) to slope downward when the total capital requirement for new projects are low since in the short-run,[7] companies tend to rely on debt for incremental capital because debt is relatively cheaper than equity and issuing equity dilutes equity ownership. But sooner or later, as more debt is raised, the adjusted WACC will move upward, as depicted in Figure 2B.

Even if the flotation costs remain constant, beyond a certain point, the WACC will increase as the debt to equity ratio (B/S) increases due to the increased level of financial risk. Note that the short-run marginal cost of capital (SRMCC) curve (i.e. the cost of debt in this case) intersects the WACC curve at its minimum point. This intersection point also identifies the optimal debt to value ratio in the short-run. However, implicit in Figure 2B is the condition that the company has found the optimal capital structure where the weighted average cost of capital equals the marginal cost of capital before the company can undertake projects up to the point where it short-run marginal return of capital equal its short-run marginal cost of capital.

B. The Long Run Solution

In the long-run, all resources/inputs are variable. Thus, the company may increase its equity capital to offset the increased level of debt as it strives to maintain the optimal (short run) debt to value ratio. Here, the long-run solution is far more complex.

Empirical research shows that other costs of issuing debt and equity (flotation costs) likely follow a step function. Because of economies of scale and bargaining power, flotation costs as a percentage will likely be lower in larger deals. Lee, Lockhead, Ritter and Zhao (1996) document that depending on the size of the deal, the total costs for an equity IPO range from 5.15% (> $500 mil.) to 15.36% ($2 mil. to $10 mil.); for secondary offerings, the costs are 3.64% (> $500 mil) to 12.88% ($2 mil. to $10 mil.); and for straight bonds, they range from 0.82% (> $500 mil) to 3.74% ($2 mil. to $10 mil.)[8] It is evident that the less complicated and the larger the deal, the lower its direct costs.

Now even if K_d and K_s are roughly constant within a certain range of capital raised, once we take into account of the lower percentage flotation costs in successive increments, the WACC curve will shift down as the amount of capital raised increases. Beyond a certain point though, the WACC curve will begin to shift upward. The reason is that when a company needs to raise successively more capital, its cost of capital (K_d and K_s) will eventually go up since the company must increase the investment return to entice ever more investors. In addition, the decline in flotation costs most likely cannot completely offset the increase in costs of debt and equity since these costs of capital account for the majority of the costs of funding.

Figure 3A illustrates the situation faced by the company in the long run. Suppose the company starts from short-run period 1, its optimal debt to value ratio is depicted as point A, the point of intersection between the company's short-run WACC ($SRWACC_1$) and its short-run marginal cost of capital curve ($SRMCC_1$). Point A also depicts the level of capital raised that minimizes the company's WACC. As the company changes its capital structure <u>and</u> the amount

[7] Short-run denotes the period when at least one of the resources is fixed.
[8] These figures were updated by Ross, Westerfield and Jordan (2006), p. 517, for the period 1990-2003.

of capital raised during the course of business in the long-run, the WACC curve and the MCC curve will shift from period 1 to period 2 and then to period 3. The WACC curve for the company in the long-run (LRWACC) is the points of tangency of the short-run WACC curves.

Following the logic espoused at the beginning of this section, in the long run when the company reaches the optimal debt to value ratio at point B, its LRWACC equals its $SRWACC_2$ and its marginal cost of capital. That is,

$$LRWACC = SRWACC_2 = SRMCC_2$$

However, before the company reaches this optimal debt to value ratio and optimal amount of capital raised in the long run (point B), the company's WACC is not equal to its marginal cost of capital. For instance, at point A the company's WACC (LRWACC) is lower than its MCC ($SRMCC_1$). Only at point B will the company's WACC equals its MCC.[9] For many companies, their practice of selecting projects by setting the project's expected marginal return against the company's WACC will lead to suboptimal results for investors.

To further clarify this point, we have a company with no capital and wants to raise $50 mil.. Assume also that the market is very liquid such that the component costs of capital remain fairly constant up to $2 bil. The tax rate is 40%. The following table shows the costs of component capital and the associated WACC.

	20% Debt	30% Debt	40% Debt
Cost of Equity	7.5%	7.7%	9.5%
Cost of Debt	10%	10.5%	12%
WACC	8.9%	8.82%	9.48%

Here, at the optimal capital structure of 30% debt, the company's WACC is 8.82%. Having successfully raised $50 mil., now the company wants to raise another $10 mil entirely in debt. Now, the marginal cost of capital for the company is 5.7% which is different from the company's WACC.[10] In the long run, the company may want to raise, say, $500 mil. Assume that the advantage of lower flotation costs alone lowers all component cost of capital by 0.2%. For example, at 20% debt level, the cost of equity is now at 7.3% and the cost of debt is at 9.8%. The associated WACCs than become 8.72% (20% debt), 8.56% (30% debt), and 9.31% (40% debt). Clearly, the company's WACC curve with different capital structures in the long run with $500 mil. of capital is lower than that in the short run in this example, similar to the conclusion drawn in Figure 3A.

C. Capital Budgeting Decision in the Long-Run

One key difference between our analysis in the short-run and the long-run is that in the long run when the company has many choices of different levels of capital raised (both debt and

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[9] As an aside, in a world with no barriers to entry, at the long run optimal level of debt to value ratio (capital structure) and optimal level of capital raised, MR=WACC=MCC since there should be zero economic profit (which includes capital costs) at this point.

[10] With an additional $10 mil. in debt, the D/V ratio = $25/$60. Hence, the MC of new capital is 9.5%(1 - 40%) = 5.7%.

equity), the company's WACC will equal its MCC only if the company reaches the point where both the debt to value ratio and the level of capital raised are optimal.
Evidently, the solution presented in Figure 2A previously is only a suboptimal short-run solution even if one assumes that the company's WACC equals it MCC. The correct solution is depicted in Figure 3A if the market is frictionless. A difficulty in making capital budgeting decision in the long-run as expressed in Figure 3A is that the overall project investments, assuming each project's risk is the same as that of the overall company, must equal the optimal level of capital raised. In fact, the company needs to determine the optimal level of capital raised before it considers the projects since the point where WACC equals MCC is jointly determined by the amount of capital raised and the debt to value ratio.

In practice, however, the optimal point, point B, in Figure 3A likely is a range as depicted by the region A to B in Figure 3B rather than a point because financial market is not frictionless.[11] Figure 3B shows that economies of scale in flotation costs and other financial costs materialize quickly before point A and diseconomies of scale do not set in until point B. Therefore, the long-run WACC is constant over a wide range of possible levels of capital raised and debt to value ratios. Between points A and B, the company's WACC equals its MCC. If the company ranks its possible projects by their respective IRRs and constructs the investment opportunity schedule (IOS), then all projects up to the point I_1 (a certain combination of capital raised and debt to value ratio) should be accepted. If a different combination of capital raised and debt to value ratio is desired, then all projects up to I_2 should be accepted. However, point I_3 is not optimal because at point I_3, the company's WACC does not equal its MCC.

V. Conclusion

This paper analyzes the relationship between the company's weighted average cost of capital and its marginal cost of capital. We show that only under certain conditions can the company's WACC equals its MCC. Microeconomic principles are used to shed insights into the well known capital budgeting rule of investing until the point where the new project's IRR equals the company's WACC. Critical to this rule is the assumption of optimal debt to value ratio (i.e. optimal capital structure). However, this paper demonstrates that even with this assumption, this common capital budgeting rule is just a suboptimal short-run solution since it does not take into account the quantity of capital being raised.

In the long-run, to minimize its WACC the company must jointly determine the optimal debt to value ratio and the optimal level of capital. Assuming that new projects have the same risk as the company's existing projects, the company can then use its WACC as the investment cutoff rate since its WACC now equals its MCC. Any violations of these conditions will lead to suboptimal investment decisions in the long-run. In practice, however, the company will probably face a range of feasible debt to value ratios and desired levels of capital due to imperfect market conditions. As a result, the long-run solution is not as restrictive as it may first appear. Our analysis of the WACC here allows practitioners to better understand the economic rationales behind the pure mathematical arguments found in the literature.

[11] For example, flotation costs as a percentage typically do not fall continuously but in steps.

Figure 1

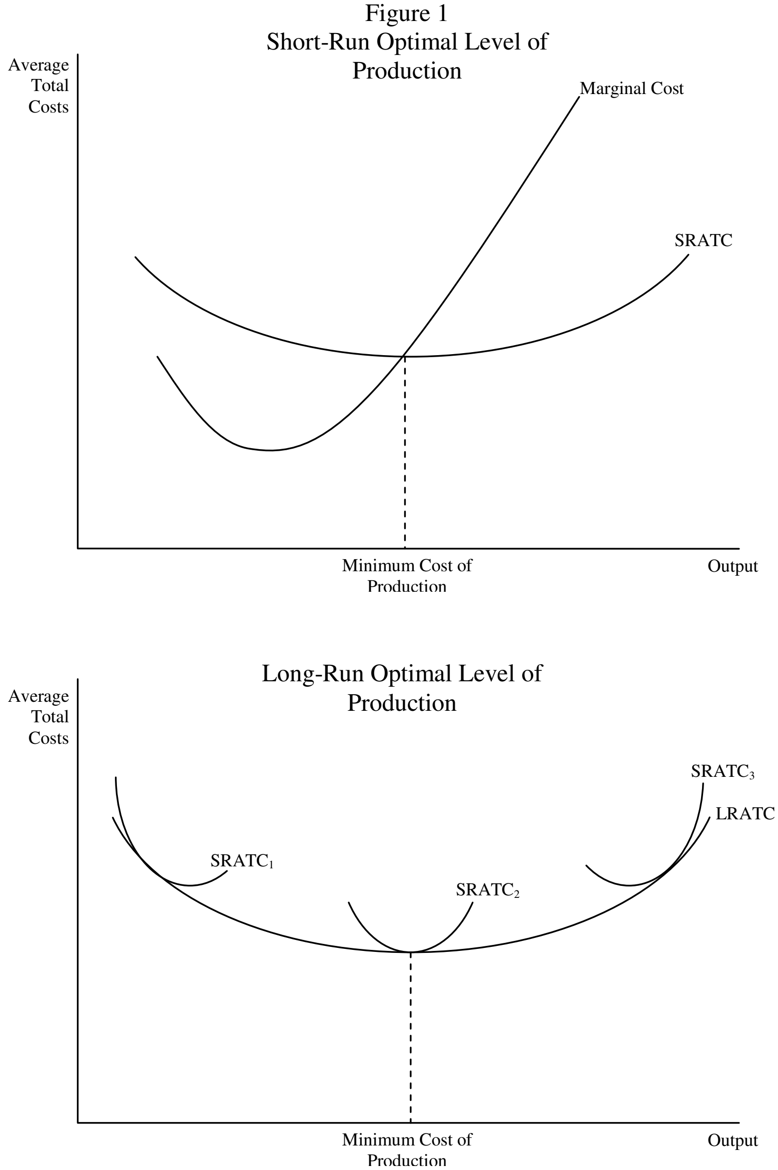
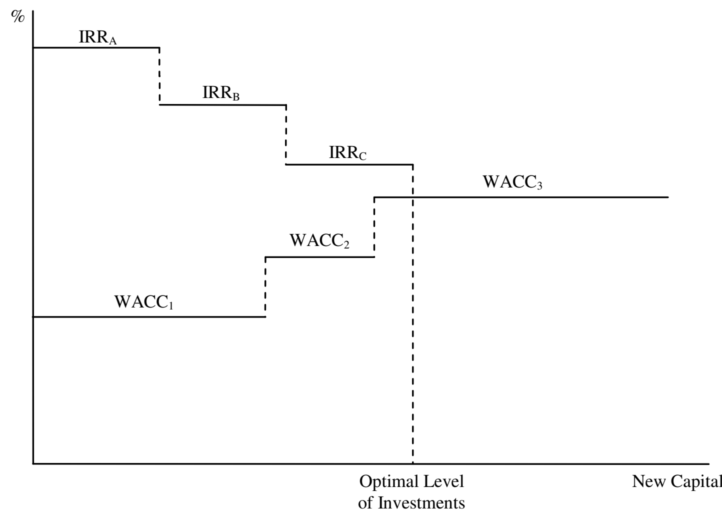

Short-Run Optimal Level of Production
Average Total Costs
Marginal Cost
SRATC
Minimum Cost of Production
Output
Long-Run Optimal Level of Production
Average Total Costs
SRATC3
LRATC
SRATC1
SRATC2
Minimum Cost of Production
Output

Figure 2A
Short-Run Marginal Cost of Capital Schedules



Note: By superimposing the investment opportunity schedule onto this figure, the company can determine the optimal level of investments in the short-run.

Figure 2B
Short-Run Optimal Capital Structure

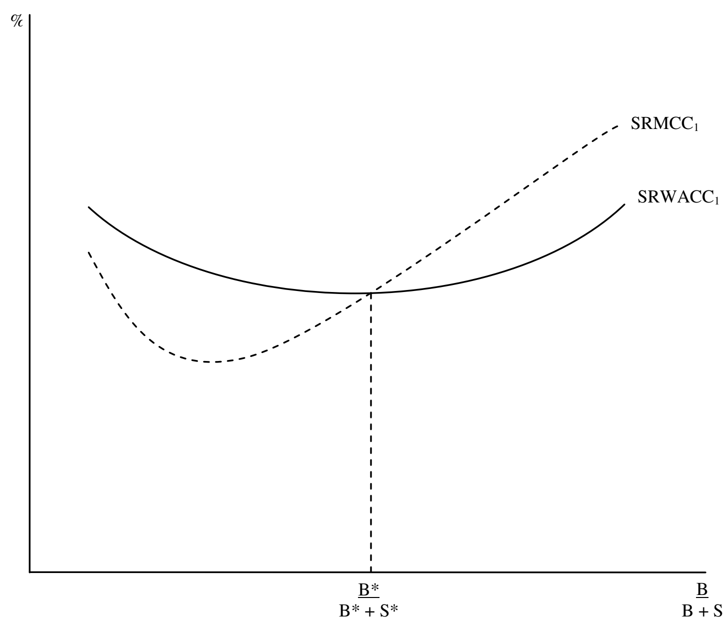


Figure 3A
Short-Run and Long-Run Optimal Level of Debt to Value Ratio and Investments

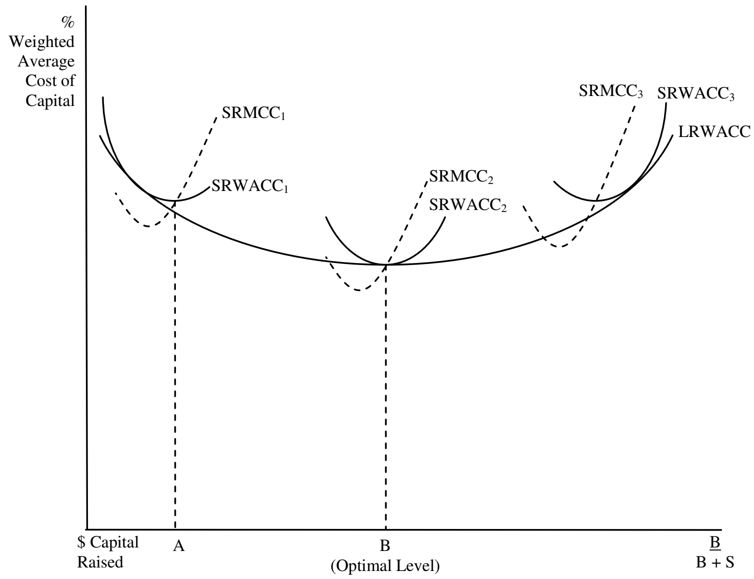
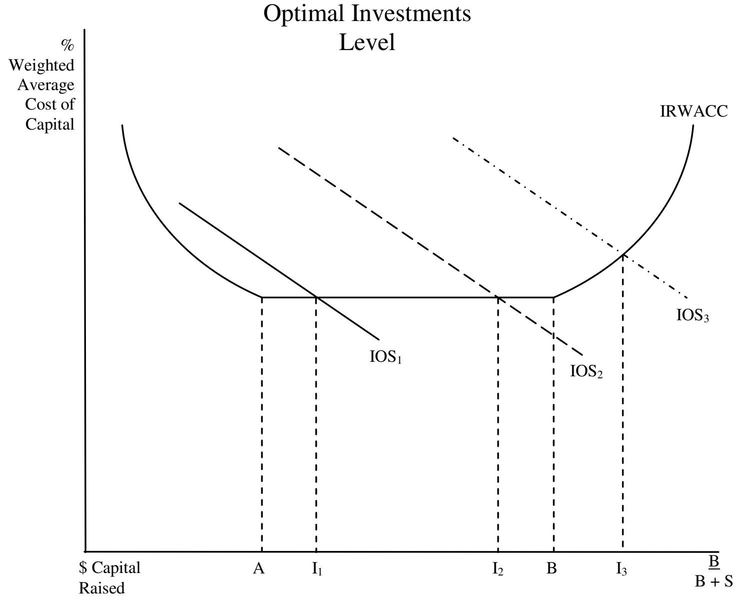


Figure 3B
Optimal Investments Level



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