CHAPTER 109

WAVE FORCES ON PIPELINES

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ABSTRACT

Water waves exert a force on a pipeline which is placed within the zone of wave influence. This force is made up of a periodic force which is composed of a drag and an inertial force, and a non-periodic second order force which acts upward.

The coefficients of mass C_M and drag C_D as defined by the "Morison equation" were evaluated for many wave and depth conditions and it was found that the coefficients vary with the wave heights, wave length, depth of the cylinder below still water surface, and within the wave cycle.

WAVE FORCE ANALYSIS

The forces exerted on a circular cylinder by unbroken surface waves, assuming the cylinder is sufficiently far from the bottom are made up of two components:

1. Inertial force which is proportional to the acceleration exerted on the mass of the water displaced by the cylinder.

2. Drag force which is proportional to the square of the water particle velocity.

In designing vertical piles it is necessary to compute only the horizontal and lift forces, as these are the only important forces acting on such a cylinder. But when designing a horizontal structural member subjected to wave action it is necessary to find the horizontal, vertical, uplift and transverse forces.

Considering a long and slender horizontal cylinder with its axis at right angles to the wave crest, only the vertical (sinusoidal), uplift and transverse force (due to eddy sheddings) are of importance. But when the cylinder is parallel to the wave crest then the vertical and horizontal components of the force become important. In addition the uplift and transverse may be of importance. For simplicity in the analysis of the data of this study, and because other theories do not necessarily yield better results, the linear theory is used.

The water surface elevation, Fig. 1, is given by:

$$\eta(x,t) = \frac{H}{2} \cos 2\pi\left(\frac{x}{L} - \frac{t}{T}\right) \qquad (1)$$

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and the water particle velocity and acceleration is given by:

$$u_{(z)} = \frac{\pi H}{T} \frac{\text{Cosh } kz}{\text{Sinh } kd} \text{ Cos } 2\pi\left(\frac{x}{L} - \frac{t}{T}\right) \tag{2}$$

$$v_{(z)} = \frac{\pi H}{T} \frac{\text{Sinh } kz}{\text{Sinh } kd} \text{ Sin } 2\pi\left(\frac{x}{L} - \frac{t}{T}\right) \tag{3}$$

$$\frac{\partial u}{\partial t}(z) = \frac{2\pi^2 H}{T^2} \frac{\text{Cosh } kz}{\text{Sinh } kd} \text{ Sin } 2\pi\left(\frac{x}{L} - \frac{t}{T}\right) \tag{4}$$

$$\frac{\partial v}{\partial t}(z) = -\frac{2\pi^2 H}{T^2} \frac{\text{Sinh } kz}{\text{Sinh } kd} \text{ Cos } 2\pi\left(\frac{x}{L} - \frac{t}{T}\right) \tag{5}$$

The center of the force meter is assumed to be fixed at x = 0, and for this location Eqs. (1) through (5) become:

$$\eta_{(t)} = \frac{H}{2} \text{ Cos } \omega t \tag{6}$$

$$u_{(z)} = \frac{\pi H}{T} \frac{\text{Cosh } kz}{\text{Sinh } kd} \text{ Cos } \omega t \tag{7}$$

$$v_{(z)} = -\frac{\pi H}{T} \frac{\text{Sinh } kz}{\text{Sinh } kd} \text{ Sin } \omega t \tag{8}$$

$$\frac{\partial u}{\partial t} = -\frac{2\pi^2 H}{T^2} \frac{\text{Cosh } kz}{\text{Sinh } kd} \text{ Sin } \omega t \tag{9}$$

$$\frac{\partial v}{\partial t} = -\frac{2\pi^2 H}{T^2} \frac{\text{Sinh } kz}{\text{Sinh } kd} \text{ Cos } \omega t \tag{10}$$

where $\omega = {}^{2\pi}/_{T}$

1. Vertical Forces

The force per unit length due to surface waves is given by the Morison Equation:

$$F_{(z,t)} = C_M V\rho \frac{\partial v}{\partial t} + \frac{1}{2} C_D \rho A v|v| \tag{11}$$

where

$V = \frac{D^2\pi}{4}$ = the volume of the displaced fluid per unit length.

$A = D$ = the projected area of the cylinder, per unit length, in the direction of flow.

v and $\partial v/\partial t$ are given by Eqs. (8) and (10). Figure (2) is a sketch showing the sign convention of η_t, drag and inertia forces and $F_{(z,t)}$. Upward force is considered positive.

Using Eqs. (8) and (10) in Eq. (11) and:

$$F_{MV} = \frac{D^2}{2} \rho \frac{\pi^3 H}{T^2} \frac{\text{Sinh } kz}{\text{Sinh } kd} \tag{12}$$

and

$$F_{DV} = \rho \frac{D}{2}\left(\frac{\pi H}{T} \frac{\text{Sinh } kz}{\text{Sinh } kd}\right)^2 \tag{13}$$

then for a horizontal cylinder parallel to the wave crest, or a very short segment of a cylinder placed at right angles to the wave crest at $x = 0$, the equation can be written:

$$- F_{(t,z)} = + C_M F_{MV} \text{ Cos } \omega t + C_D F_{DV} \text{ Sin } \omega t \left|\text{Sin } \omega t\right| \tag{14}$$

Equation (14) and Fig. 2 show that the Maximum downward force occurs at $0 \leq t \leq \frac{T}{4}$, and the maximum upward force occurs at $\frac{T}{2} \leq t \leq \frac{3T}{4}$.

2. Horizontal Forces

By a similar analogy, the Morison Equation for a force acting horizontally on a horizontal cylinder with its axis parallel to the wave crest is:

$$F_{(z,t)} = - C_M F_M \text{ Sin } \omega t + C_D F_D \text{ Cos } \omega t \left|\text{Cos } \omega t\right| \tag{15}$$

where

$$F_M = \frac{\pi^3 D^2}{2} \rho \frac{H}{T^2} \frac{\text{Cosh } kz}{\text{Sinh } kd} \tag{16}$$

and

$$F_D = \rho \frac{D}{2}\left(\frac{\pi H}{T} \frac{\text{Cosh } kz}{\text{Sinh } kd}\right)^2 \tag{17}$$

Figure 3 is a sketch showing the sign convention of $\eta(t)$, drag and inertial forces and $F_{(z,t)}$ for the horizontal forces. Force is positive in the direction of wave advance. Equation (15) and Fig. 3 show that the maximum horizontal component force in the direction of wave propogation occurs when $\frac{3}{4}T \leq t \leq T$ and the maximum negative force occurs at $\frac{1}{4}T \leq t \leq \frac{1}{2}T$.

Examining Eqs. (20) and (21) it can be seen that the maximum horizontal force occurs at $\frac{T}{4}$ seconds after the maximum vertical force (both in the positive and negative directions), assuming the coefficients of drag and mass (C_D and C_M) remain constant throughout the wave cycle.

3. Coefficients of Mass and Drag (C_M and C_D)

One of the problems in the design of structural elements to withstand the dynamic forces of waves is to find and use the appropriate values of C_M and C_D.

The compute C_M and C_D from force measurements consider Eq. (14) which can be written (for $t \leq \frac{T}{4}$) :

$$- F_{(z,t)} = C_M F_{MV} \cos \omega t + C_D F_{DV} \sin^2 \omega t \tag{18}$$

The maximum downward force occurs at $\dfrac{\partial -F_{(z,t)}}{\partial t} = 0$

That is, $C_M F_{MV} \omega \sin \omega t = 2C_D F_{DV} \omega \sin \omega t \cos \omega t$

$$\cos \omega t_1 = \frac{C_M F_{MV}}{2C_D F_{DV}} \tag{19}$$

t_1 is the time phase of the maximum downward force relative to the crest, that is the maximum downward force leads the crest by t_1. Therefore the maximum force is given by Eq. (18) as evaluated at t_1. That is:

$$- F_{(z,t_1)} = C_M F_{MV} \cos \omega t_1 + C_D F_{DV} \sin^2 \omega t_1 \tag{20}$$

which could be written as

$$- F_{(z,t_1)} = \frac{C_M^2 F_{MV}^2 + 4 C_D^2 F_{DV}^2}{4 C_D F_{DV}} \tag{21}$$

This procedure assumes that C_M and C_D are constants, at least for $0 \leq t \leq \frac{T}{4}$.

Solving Eqs. (19) and (20) for C_M and C_D gives:

$$C_M = \frac{-2\ F_{(z,t_1)}}{F_{MV}(1 + \text{Cos}\ \omega t_1)} \tag{22}$$

$$C_D = \frac{-F_{(z,t_1)}}{F_{DV}(1 + \text{Cos}\ \omega t_1)} \tag{23}$$

Similarly for the horizontal force, consider Eq. (15) at $\frac{T}{4} \leq t \leq \frac{T}{2}$ which becomes:

$$-\ F_{(z,t)} = C_M\ F_M \text{Sin}\ \omega t + C_D\ F_D \cos^2 \omega t \tag{24}$$

for maximum force

$$\frac{\partial\ (-F_{(z,t)})}{\partial\ t} = 0$$

that is:

$$C_M\ F_M\ \omega \text{Cos}\ \omega t = 2C_D\ F_D\ \omega \text{Sin}\ \omega t\ \text{Cos}\ \omega t$$

$$\text{Sin}\ \omega t_1 = \frac{C_M\ F_M}{2C_D\ F_D} \tag{25}$$

t_1 here is the time phase of the maximum negative horizontal force relative to the crest. The maximum negative force is then given by:

$$-\ F_{(z,t_1)} = C_M\ F_M\ \text{Sin}\ \omega t_1 + C_D\ F_D\ \text{Cos}^2\ \omega t_1 \tag{26}$$

or

$$-\ F_{(z,t_1)} = \frac{C_M^2\ F_M^2 + 4C_D^2\ F_D^2}{4C_D\ F_D} \tag{27}$$

Here C_D and C_M are assumed constant for $\frac{T}{4} \leq t \leq \frac{T}{2}$. Solving Eqs. (25) and (26) for C_M and C_D gives:

$$C_M = \frac{-2\, F_{(z,t_1)} \operatorname{Sin} \omega t_1}{F_M(1 + \operatorname{Sin}^2 \omega t_1)} \tag{28}$$

$$C_D = \frac{-F_{(z,t_1)}}{F_D(1 + \operatorname{Sin}^2 \omega t_1)} \tag{29}$$

Computing C_M at t=0 and $t = \frac{T}{2}$ and C_D at $t = \frac{T}{4}$ and $\frac{3}{4}T$ does not require making the assumption that C_M and C_D are time independent. C_M and C_D are empirical coefficients, they were found to be a function of t, by Keulegan and Carpenter (1958), and of z, by Jen (1968).

4. C_M and C_D as functions of time.

Consider a case in which the time history of the wave, $\eta_{(t)}$, and the time history of the force, $F_{(z,t)}$ are known (Eqs. 6 and 14). Then for each time, t, we have an equation for C_M and C_D of the form:

$$A = B \cdot C_M + E \cdot C_D \tag{30}$$

where A, B and E are constants and could be computed from the time history of the wave characteristic and the cylinder diameter. Therefore if Eq. 26 is evaluated at t_1, t_2, t_3, t_4...etc. and any two consecutive equations are solved simultaneously, vales of C_M and C_D at $t = \frac{t_1 + t_2}{2}$ etc. could be computed. These will show any dependence of C_M and C_D on time within the wave cycle.

5. Force on an Inclined Cylinder.

Consider a cylinder inclined at an angle θ to the vertical, in a vertical plane that is normal to the wave crests, Fig. 4. The velocity and acceleration components which exert the maximum force on the cylinder at any time, t, are those which are at right angles to the cylinder.

From Eqs. (7) and (8) the component of water particle velocity (R_u) perpendicular to the cylinder becomes:

$$R_u = \frac{\pi H}{T^2} \frac{1}{\operatorname{Sinh} kd} \left\{ \operatorname{Cosh} kz \operatorname{Cos} \omega t \operatorname{Cos} \theta - \operatorname{Sinh} kz \operatorname{Sin} \omega t \operatorname{Sin} \theta \right\} \tag{31}$$

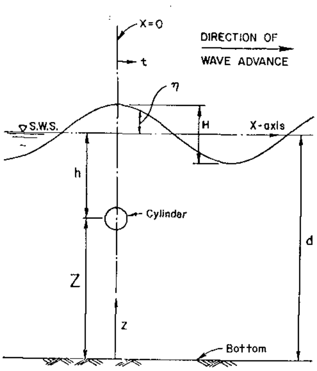


Fig. 1 Definition Sketch

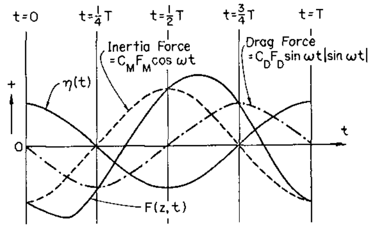


Fig. 2 Sign Convention of $\eta(t)$, and the Vertical Forces

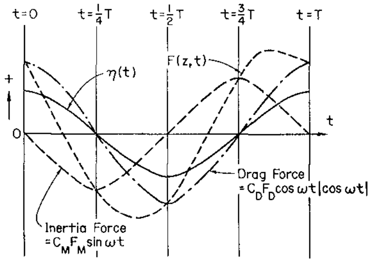


Fig. 3 Sign Convention of $\eta(t)$ and the Horizontal Forces

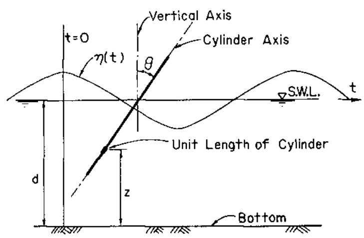


Fig. 4 Inclined Cylinder Configuration

Similarly the perpendicular acceleration (R_A) obtained from Eqs. (9) and (10) is:

$$R_A = \frac{-2\pi^2 H}{T^2} \frac{1}{\text{Sinh } kd} \left\{ \text{Cosh } kz \text{ Sin } \omega t \text{ Cos}\theta + \text{Sinh } kz \text{ Cos } \omega t \text{ Sin}\theta \right\} \quad (32)$$

Using Eqs. (22) and (23) in Eq. (11) for $0 \le t \le \frac{T}{4}$ one can obtain the force per unit length acting on the inclined cylinder, $F(z,t,\theta)$.

$$F_{(z,t,\theta)} = + C_M \left\{ F_M \text{ Cos } \theta \text{ Sin } \omega t + F_{MV} \text{ Sin } \theta \text{ Cos } \omega t \right\}$$

$$+ C_D \left\{ F_D \text{ Cos}^2 \theta \text{ Cos}^2 \omega t + F_{DV} \text{ Sin}^2 \theta \text{ Sin}^2 \omega t \right.$$

$$\left. - \tfrac{1}{2}\sqrt{F_D \cdot F_{DV}} \quad \text{Cos}2\theta \text{ Sin } 2 \omega t \right\} \quad (33)$$

F_M, F_{MV}, F_D and F_{DV} have been defined previously. In Eq. (33) $F_{(z,t,\theta)}$ is considered positive if its horizontal component is in the direction of wave advance and its vertical component is downwards.

Assuming deep water conditions (i.e., $\frac{d}{L} < 0.5$), Cos kz $\rightarrow$ Sinh kz, $F_M \rightarrow F_{MV}$; and $F_D \rightarrow F_{DV}$. Then Eq. (8) becomes

$$F_{(z,t,\theta)} = C_M F_M \text{ Sin}(\omega t + \theta) + C_D F_D \text{ Cos}^2 (\omega t + \theta) \quad (34)$$

For maximum Force:

$$\frac{\partial F}{\partial t} (z,t,\theta) = C_M F_M \omega\text{Cos}(\omega t + \theta) - 2C_D F_D \omega\text{Sin}(\omega t + \theta)\text{Cos}(\omega t + \theta) = 0$$

therefore

$$\text{Sin}(\omega t_1 + \theta) = \frac{C_M F_M}{2C_D F_D} \quad (35)$$

Thus the maximum force occurs at t_1 when Eq. (35) is satisfied, and it is:

$$F_{(z,t_1,\theta)\max} = C_M F_M \text{ Sin}(\omega t_1 + \theta) + C_D F_D \text{ Cos}^2(\omega t_1 + \theta) \quad (36)$$

Substitution Eq. (35) in Eq. (36) one can obtain

$$F_{(z,t_1,\theta)} + \frac{C_M^2 F_M^2 + 4C_D^2 F_D^2}{4C_D F_D} \quad (37)$$

From Eqs. (35) and (36) it is seen that the magnitude of the maximum force per unit length on an inclined cylinder is independent of the inclination angle θ; that is, it is equal for all values of θ. The only thing that changes with the angle of inclination is the time phase at which the maximum force occurs. In shallow water conditions that maximum force per unit length occurs when the cylinder is vertical, and in any other position the force will be smaller. This could be explained by the orbital path of the water particles, which is elliptic.

UPLIFT FORCES

A non-periodic upward force was observed during the early stages of the laboratory experiments. When a wave passed over a horizontal cylinder, rigidly fixed at right angles to the wave crest, a constant upward force was produced.

Explanation

The Stokes second order equation for pressure is given by:

$$\frac{P}{\rho g} + y = \frac{H}{2} \frac{\text{Cosh } kz}{\text{Cosh } kd} \text{ Cos } 2\pi \left(\frac{x}{L} - \frac{t}{T}\right)$$

$$+ \frac{3}{8} \frac{\pi H^2}{L} \frac{\text{Tanh } kd}{\text{Sinh}^2 kd} \frac{\text{Cosh } 2kz}{\text{Sinh}^2 kd} - \frac{1}{3} \text{ Cos } 4\pi\left(\frac{x}{L} - \frac{t}{T}\right)$$

$$- \frac{1}{8} \frac{\pi H^2}{L} \frac{\text{Tanh } kd}{\text{Sinh}^2 kd} \text{ Cosh } 2kz \qquad (38)$$

where: P is the pressure at any point x,y in the fluid and all other symbols have been defined previously, (Wiegel, 1964).

The mean pressure throughout the wave cycle is given by:

$$p = \frac{1}{T} \int_0^T \left(\frac{P}{\rho g} - y\right) dt$$

Therefore

$$p = \frac{1}{8} \frac{\pi H^2}{L} \frac{\text{Tanh } kd}{\text{Sinh}^2 kd} \text{ Cosh } 2kz \qquad (39)$$

As the pressure in a fluid is the same in all direction, and acts normal to the cylinder, the uplift force per unit length on a circular cylinder, mounted horizontally and parallel to the wave crest, is given by integrating the vertical component of the pressure as given in Eq. (39) around the circumference of the cylinder. See Fig. 5. Therefore:

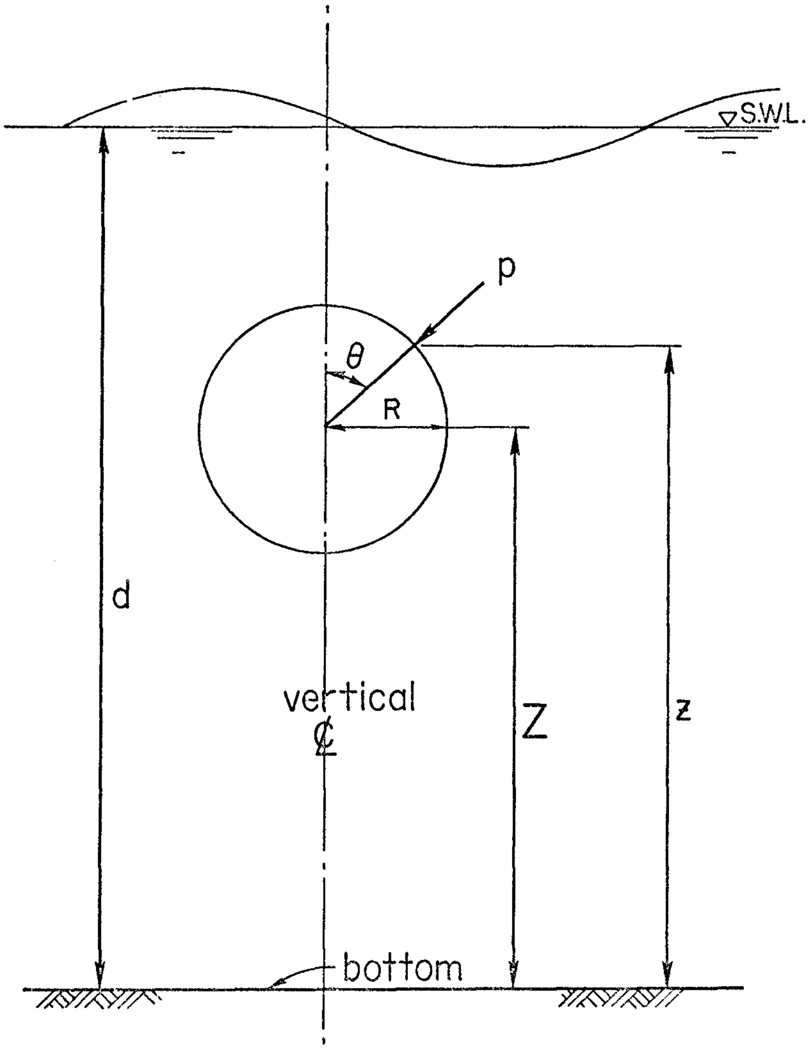


Fig. 5 Definition Sketch Showing Pressure on the Cylinder

$$\text{uplift} = \frac{1}{8}\ \frac{\pi H^2}{L}\ \frac{\text{Tanh kd}}{\text{Sinh}^2\text{kd}} \cdot 2 \int_0^\pi \text{Cosh } 2kz \text{ Cos}\theta \cdot R d\theta$$

and $z = Z + R\text{Cos}\theta$ (see Fig. 5)

Therefore:

$$\text{uplift} = \frac{1}{4}\ \frac{\pi H^2}{L}\ \frac{\text{Tanh kd}}{\text{Sinh}^2\text{kd}} \int_0^\pi R\text{Cos}\theta \text{ Cosh } 2k(Z + R\text{Cos}\theta)\ d\theta$$

Then the uplift per unit length is reduced to:

$$\text{uplift} = \frac{1}{8}\ \frac{\pi^2 H^2}{L}\ \frac{\text{Sinh } 2kZ}{\text{Sinh } 2kd} \cdot R \cdot I_1\ (2kR) \tag{40}$$

The units of this uplift is in unit of pressure head. To change it to units of force it should be multiplied by ρg, then the uplift equation becomes:

$$\text{uplift/unit length} = \frac{1}{8}\ \rho g\ \frac{\pi^2 H^2}{L}\ \frac{\text{Sinh } 2kZ}{\text{Sinh } 2kd} \cdot R \cdot I_1\ (2kR) \tag{41}$$

where:

R = radius of the cylinder.

Z = the distance between the center of the cylinder and the sea floor.

I_1 (2kR) = Bessel's Function of the first kind.

When all the lengths are measured in feet and ρg is taken to equal 62.4 lb/ft^3, the uplift force is then given in lb/ft. Equation (40) is true also for a cylinder fixed at right angles to the wave crest.

DATA ANALYSIS

Data:

Three force meters were used in this study. Two of the force meters were mounted horizontally, parallel to the wave crest. One of these was 1.5 inch O.D. circular cylinder and the other was a 4.0 inch O.D. circular cylinder; both were 11.25 inches long. 606 runs were made with the 1.5 inch O.D. force meter, forces being measured only in the vertical direction. The force meter was mounted at various depths beneath the still-water surface between 3.5 inches and 19.31 inches, for wave periods varying from 0.47 sec. to 1.32 secs. and wave heights from 0.075 ft. to 0.0160 ft. With the 4 inch O.D. force meter 934 runs were made to measure forces in the vertical direction and 755 runs to measure forces in the horizontal direction. The depth of the cylinder below the still-water surface was varied from 4.42 inches to 22.88 inches, the wave period from 0.47 sec. to 1.33 secs., and the wave height from 0.08 ft. to 0.215 ft.

Figure 6 shows a sample record obtained using the 1.0 inch segment, 2.0 inches O.D. force meter (shown in Fig. (7)) which was placed at right angles to the wave crest. Six sets of records were made with the force meter located at distances from 3.0 inches to 17.0 inches beneath the still-water surface. Each set consisted of many runs, with wave periods varying from 0.41 sec. to 1.32 secs., and wave heights from 0.067 ft. to 0.243 ft. Eight sets of records were also made with the last mentioned force meter placed at a number of different angles to the horizontal (Fig. 4). The axis of the cylinder was always in a plane at right angles to the wave crest. Each of these sets consist of 33 runs.

Data Reduction

1. Sinusoidal Wave Force - Mass and Drag Coefficients:

Two methods were used to compute values of the coefficient of mass and the coefficient of drag at specific points in the wave cycle. The first was to evaluate C_M and C_D at the time when the maximum force occurred. The other was to evaluate C_M when the vertical or horizontal component of the water particle velocity was zero (depending on whether the vertical or horizontal forces were being considered) and C_D when the vertical or horizontal component of the acceleration was zero.

To evaluate C_M and C_D at the time of the maximum force, the wave periods (T), the wave heights (H), the maximum force (F_{t_1}), and the phase lag (t_1) at which this maximum force occurred were found from the records of the experiments. Then C_M and C_D are evaluated from the vertical force records using Eqs. (22) and (23). Similarly C_M and C_D are computed from the horizontal force records using Eqs. (28) and (29).

To evaluate C_M and C_D at the zero horizontal or vertical component of water particle velocity and acceleration, respectively, the maximum inertia force (I.F.) and the maximum drag force (D.F.) were measured from the oscilograph records. Then using Eqs. (18) and (15) C_M and C_D can be evaluated as follows, for vertical forces:

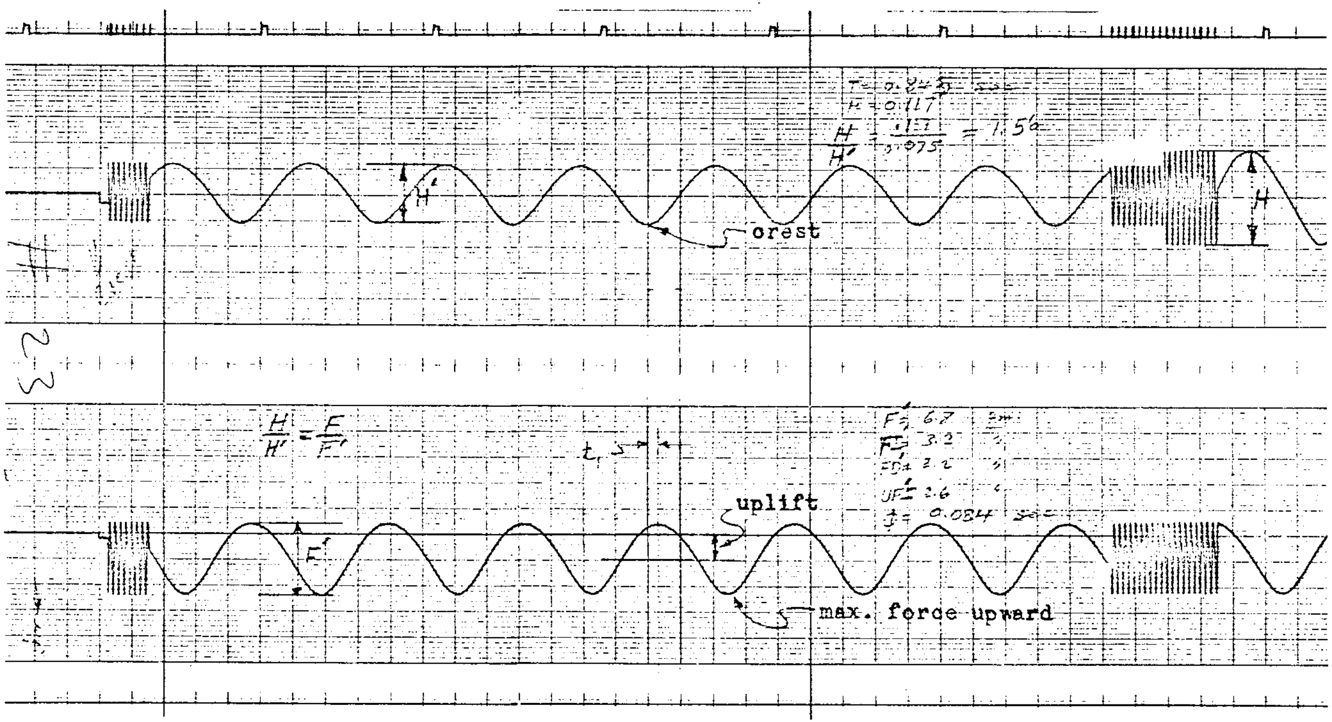
$$C_M = \frac{I.F.}{F_{MV}} \quad \text{and} \quad C_D = \frac{D.F.}{F_{DV}} \tag{42}$$

and for horizontal forces:

$$C_M = \frac{I.F.}{F_M} \quad \text{and} \quad C_D = \frac{D.F.}{F_D} \tag{43}$$

C_M and C_D were evaluated by these two methods for force meter diameters of 4.0 inches and 2.0 inches, for many wave conditions.

C_M and C_D were also evaluated at regular intervals throughout the wave cycle for one set of records. A sample plot of C_M and C_D as they vary within the wave cycle is shown in Figs. 8 and 9.



T = 0.85 sec., H = 0.117 ft., F = 10.95 gm., t_{11} = 0.08 sec., IF = 5.15 gm., DF = 3.54 gm., Uplift = 2.6 gm., Cylinder O.D. = 2.0 in., d = 2.13 ft., h = 3.0 in., Cylinder length = 1.0 in. (Set № 1, Record № 6, Run № 2)

FIG. 6 SAMPLE RECORD – VERTICAL FORCE

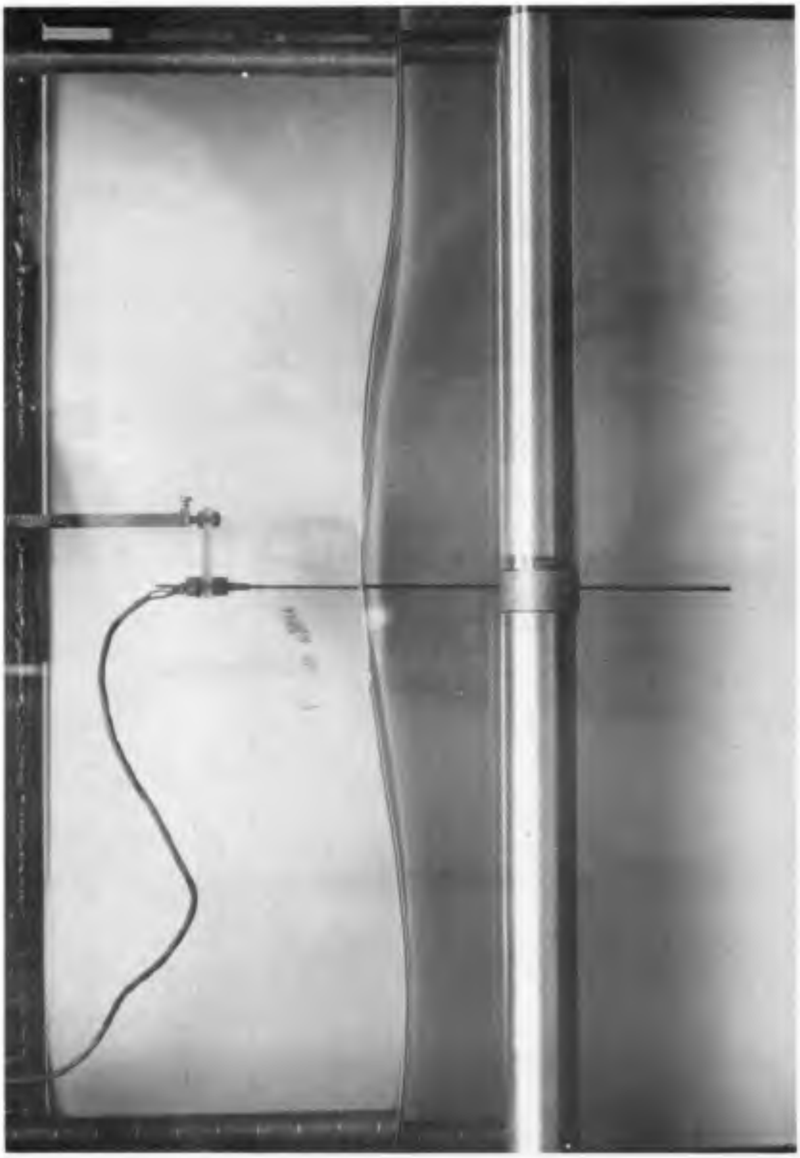


FIG. 7 FORCE METER

RESULTS AND DISCUSSION

The Coefficients of Mass and Drag

The coefficient of mass, C_M, and the coefficient of drag, C_D, were computed at two points in the wave cycle for the 4.0 inch O.D. cylinder, both for horizontal and vertical forces, and for the 2.0 inch O.D. cylinder placed at right angles to the wave crest. The two specific points are at the time when the maximum force occurs, and when the drag force or the inertial force is zero.

The following characteristics were observed for C_M:

1. C_M decreases as the wave height increases, if the wave period remains the same. This agrees with the results obtained by Evans (1970).

2. C_M increases as the depth of the cylinder beneath the still-water surface increases, if the wave characteristics remain the same. This differs from the findings of Shell Oil Company who conducted an analysis on wave and force data obtained in the Gulf of Mexico, on a vertical pile. They found that C_M decreased as the distance beneath the still-water surface increased (Evans, 1970).

3. C_M increases as the wave period, T, increases, if the wave height remains the same.

4. C_M computed from the vertical forces on the 4.0 inch O.D. cylinder was slightly smaller than that computed from the horizontal forces for the same wave conditions.

However, a definite relationship seems to exist between C_M and D_{pV}, the ratio of the vertical diameter of the orbit of the water particle to the diameter of the cylinder, this relationship is shown in Figure 10. A similar relationship exists between C_M and D_{pH}, the ratio of the horizontal diameter of the orbit of the water particle to the diameter of the cylinder, D.

2. Uplift Forces (None-Periodic)

The non-periodic uplift force was measured for all runs when the cylinder was at right angles to the wave crest. (No uplift force was detected for cylinders parallel to the wave crest.) Then the uplift force per unit length of cylinder was computed from Eq. (41). The computed uplift force was found to be always smaller than the measured force. An empirical coefficient, C, was then obtained by dividing the measured uplift force by the computed value. This coefficient was then plotted against the wave steepness, (H/L). Figure 11 shows this relationship.

Thus, the uplift equation becomes:

$$\text{Uplift} = \frac{1}{8}C \cdot \rho g \, \frac{\pi^2 H^2}{L} \, \frac{\text{Sinh } 2kx}{\text{Sinh } 2kd} \cdot R \cdot I_1 \, (2kR) \qquad (44)$$

This uplift was also evaluated for all runs made with the 2 inch O.D. force meter.

The uplift force was also evaluated using Goodman's equation for uplift which is:

$$\text{Uplift}_G = \frac{1}{2}\pi^3 \frac{\rho H^2 D^2}{T^2} k e^{-2kh} \alpha_1(kR) \quad (45)$$

where h is the depth of the cylinder center line beneath the still water surface, R is the radius of the cylinder and $\alpha_1(kR)$ is evaluated for each kR. This uplift was also smaller than the measured values. A coefficient (C_g) was obtained by dividing the measured uplift by that computed using Eq. (45) and it was plotted against H/L, Figure 12. The relationship was found to be similar to that between C and H/L. The modified Goodman's Eq. (45) becomes

$$\text{Uplift}_G = \frac{1}{2} C_g \pi^3 \rho \frac{H^2 D^2}{T^2} k e^{-2kh} \alpha_1(kR) \quad (46)$$

3. Forces on an Inclined Cylinder

The maximum force on an included circular cylinder for a specific wave was obtained from the oscillograph records. The maximum sinusoidal force for a specific wave length and wave height was plotted against the angle of inclination of the cylinder to the vertical. Figure 13 shows such plots. (Here, WH = wave height and WL = wave length.)

It appears on theoretical grounds that, for deep water waves (circular water particle paths), the maximum force on a unit length of cylinder is not changed in magnitude by changing the angle of inclination of the cylinder, if the wave characteristics remain constant, and if that unit length of cylinder remains at the same depth below the still-water surface. However the time phase at which the maximum force occurs does change. Figure 14 shows the change of t_1/T with the angle of inclination of the cylinder with the vertical (θ) as defined in Figure 4.

CONCLUSIONS

The following conclusions are drawn from this study:

1. The maximum force, time phase solution for C_M and C_D, which is defined by the Morison equation, is a useful method of evaluating C_M and C_D for a cylinder subjected to wave action as it yields the values of C_M and C_D at a time of maximum force. This value of CM is larger than that computed when the drag force is zero in both the vertical and horizontal direction.

2. Once CM and CD are found the sinusoidal force amplitude can be computed accurately from 21 for the vertical force, and from 27 for the horizontal force.

3. The coefficient of mass, CM, is not constant and it varies with the wave height, wave period, depth of cylinder beneath the still-water surface and the diameter of the cylinder. It also varies within the wave cycle when all the previously mentioned variable are held constant. CD also appears to vary with wave height, wave period, cylinder depth below still water level and the diameter of the cylinder. It also varies within the wave cycle.

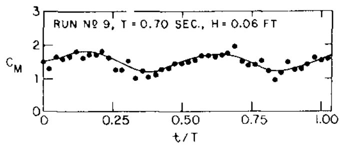


FIG. 8 COEFFICIENT OF MASS C_M VERSUS t/T

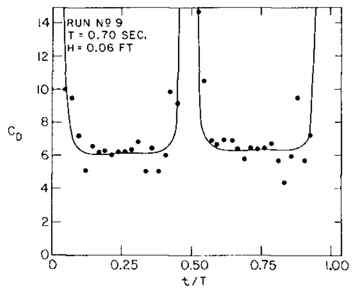


FIG. 9 COEFFICIENT OF DRAG C_D VERSUS t/T

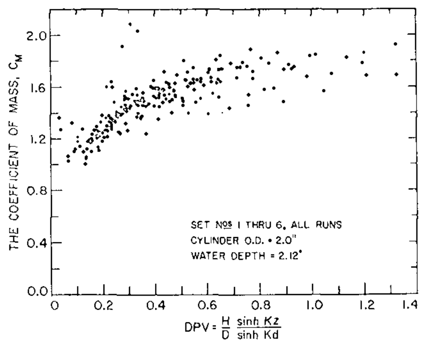


FIG. 10 THE COEFFICIENT OF MASS VS THE RATIO OF ORBITAL PATH DIAMETER TO THE O.D. OF THE CYLINDER (DPV)

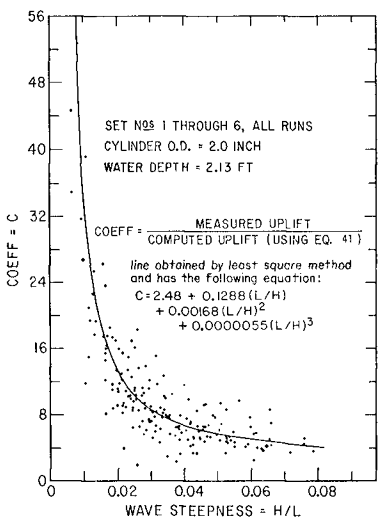


FIG. 11 UPLIFT COEFFICIENT (COEFF) VS WAVE STEEPNESS

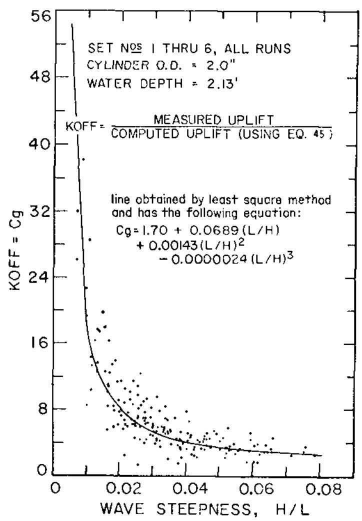


FIG. 12 UPLIFT COEFFICIENT (KOFF) VS WAVE STEEPNESS, H/L

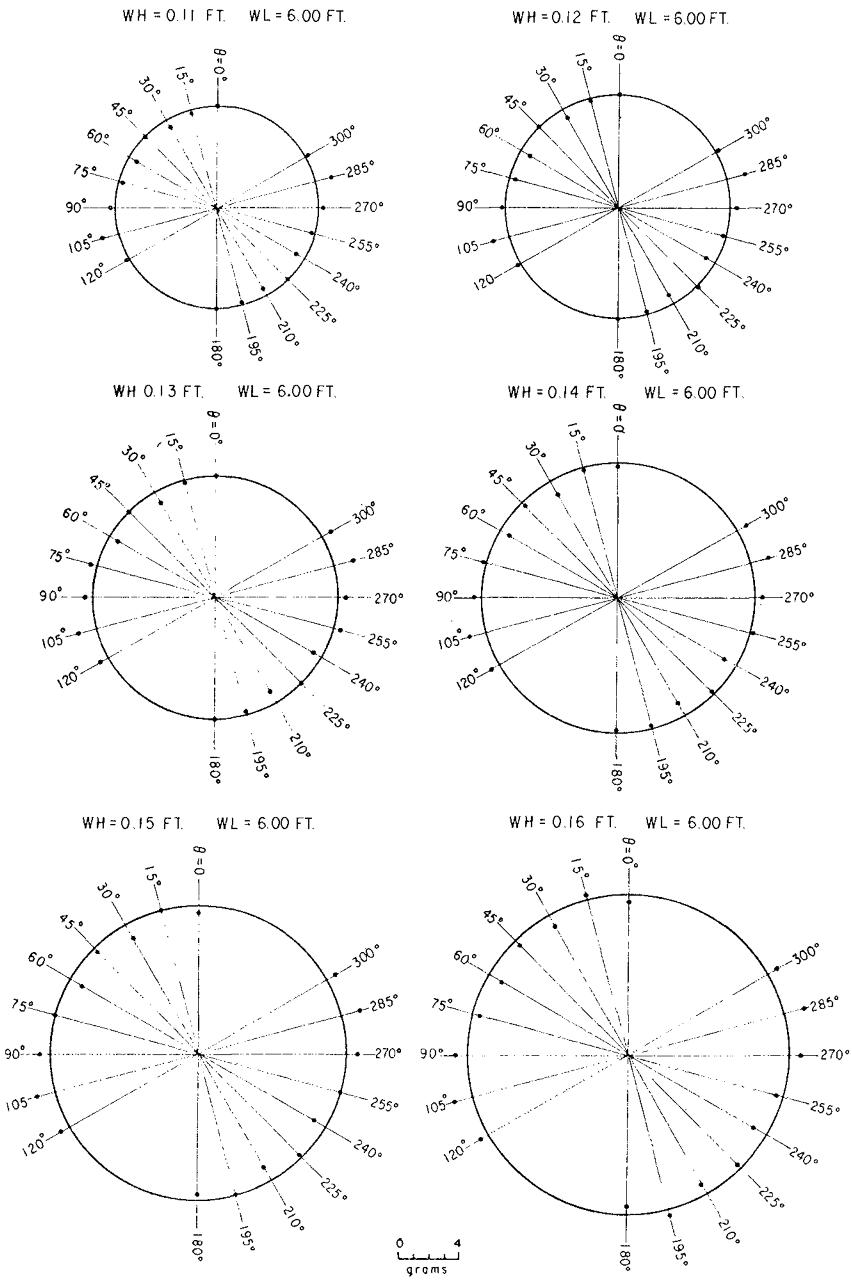


FIG. 13 FORCE VS. ANGLE OF INCLINATION

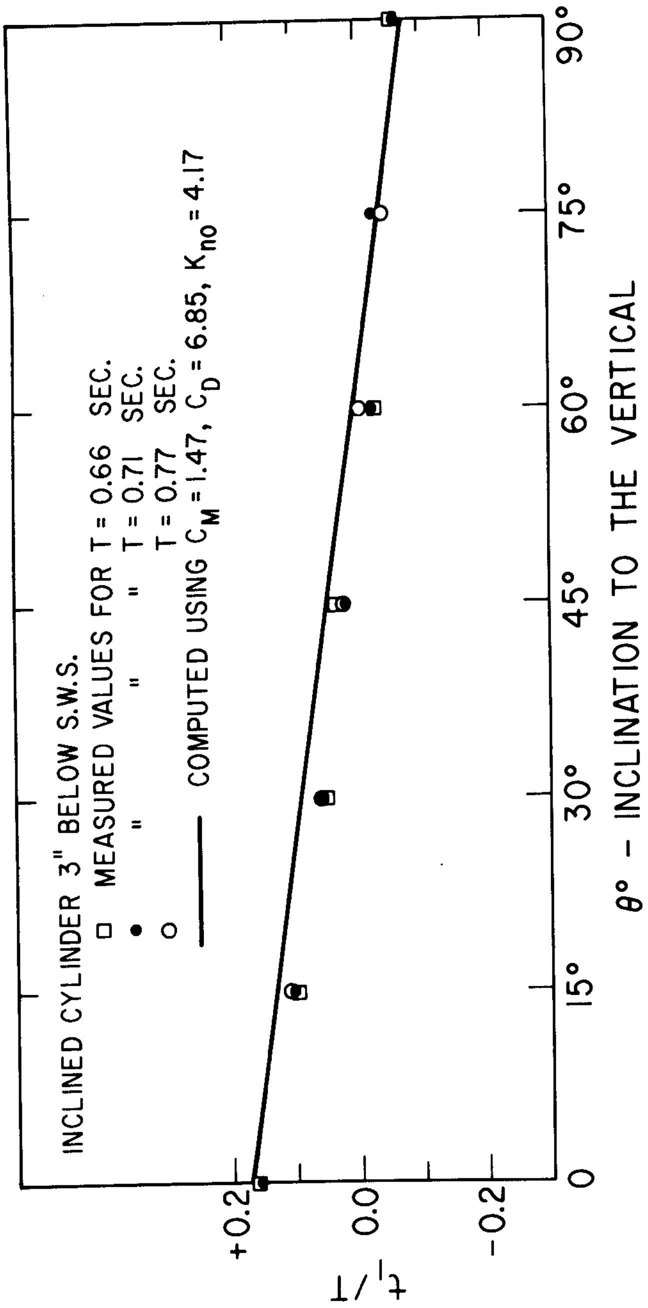


FIG. 14 DIMENSIONLESS TIME PHASE VS. INCLINATION ANGLE

4. The method used to evaluate CM and CD within the wave cycle is a very useful method to show the variation of C_M and C_D within the wave cycle.

5. The coefficient of mass, CM, varies with the time in the wave cycle in the shape of a sinusoid with frequency twice that of the wave causing it. CM is always positive.

6. When the cylinder is at right angles to the wave crest, it experiences a non-periodic uplift force of appreciable magnitude.

7. The magnitude of this uplift force could be estimated reasonably well using Eqs. (44) and (46), provided the empirical coefficient C or Cg are known.

8. For the cases studied the coefficients C and Cg seem to have a definite relationship with the wave steepness.

9. For deep-water conditions ($d > L/2$) the magnitude of the sinusoidal force amplitude does not change when the inclination of the cylinder to the horizontal is changed, but the phase time at which this maximum force occurs, relative to the wave crest, does change.

10. $A_{max}T^2/D$ appears to be a useful parameter to correlate with CM.

11. The ratio of the diameter of the water particle orbital path to the cylinder diameter, $(\frac{H}{D} \frac{\sinh kz}{\sinh kd})$ in the vertical direction or $(\frac{H}{D} \frac{\cosh kz}{\sinh kd})$ in the horizontal direction also appears to be a useful parameter to correlate with CM.

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