Consecutive shock waves and fatigue loads: action invariants as optimization parameters under Laser Shock Peening

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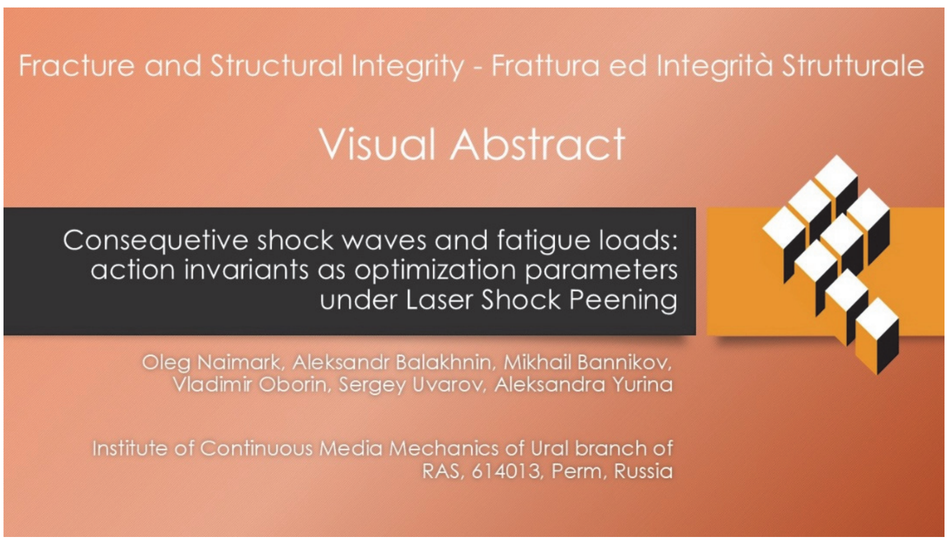


Citation: Naimark, O., Balakhnin, A., Bannikov, M., Oborin, V., Uvarov, S., Yurina, A., Consecutive shock waves and fatigue loads: action invariants as optimization parameters under Laser Shock Peening, Fracture and Stuctural integrity, 752026) 250-264

Received: 04.09.2025
Accepted: 30.10.2025
Published: 02.11.2025
Issue: 01.2026

KEYWORDS. Multiscale damage kinetics, Very High Cycle Fatigue, Laser Shock Peening, Fatigue crack kinetics, Action invariants, Self-similarity, Quantitative fractography.

INTRODUCTION

The shock wave (SW) loading of metallic materials induced by laser shock peening (LSP) is a promising method for processing near-surface layers that are most susceptible to fatigue and Foreign Object Damage (FOD) in aerospace and aircraft constructions [1, 2, 3, 4]. SWs are generated by a high-energy laser pulse of nanosecond duration (energy 1–15 J, duration <20 ns) under conditions of ablation in a metal foil applied to the material being processed and during the formation of a plasma torch initiating a shock pulse of compression. An increase in fatigue life and dynamic strength in a layer of several millimeters occurs in the SW fronts with duration of 5–20 ns and at strain rates in the range of 10^6–10^9 s^{-1}.

A shock wave pulse with pressure amplitude exceeding the Hugoniot elastic limit (HEL) is responsible for structure formation in intense plastic deformation fields that can affect the fatigue properties and dynamic resistance of materials during the operation cycle.
In general, the experiments demonstrate the complexity of material loading under LCP conditions, which is due to the unique deformation and fracture mechanisms that occur during shock-wave loading, when the structural relaxation mechanisms characteristic of both ductile and brittle fractures are consistently realized in compression and rarefaction waves. In this case, it is of fundamental importance to establish the relationship between the regularities of elastic-plastic front formation and the structurally determined relaxation mechanisms and, as a consequence, the microstructure of metals after the passage of a shock-wave pulse. It is also essential to exclude situations accompanied by the formation of localized damage preceding spalling, which can lead to critical situations in the flight cycle due to a sharp decrease in the fatigue life. To solve this problem, it is necessary to conduct fundamental research on the relationship between the patterns of an elastic-plastic front and the changes in material structure and compare the results with experiments on shock-wave loading of materials performed using the LSP method and ballistic set-ups with in situ registration of the parameters of a shock-wave pulse by the Doppler interferometry (VISAR) method [5].
Optimization of laser shock-wave modes as applied to the processing of materials and structures involves a methodological support for two modes of material loading: laser-driven shock-waves, and subsequent loading simulating operational responses. Taking into account the characteristic parameters of shock-wave loading under LSP (amplitudes of ~ 1-10 GPa) [1,6-8], parameters that provide links between the structural changes in a shock wave and assessment of the fatigue life should be based on invariant characteristics that show how material structure changes.
The aim of this research is to justify the application of invariant parameters for both types of loading and to use this approach to optimize LSP modes for aircraft engine alloys. The framework of the study is built upon the results of original experiments and modeling of the behavior of materials under successive dynamic (shock-wave) and high cycle fatigue (HCF) and very high cycle fatigue (VHCF). This made it possible to establish a correlation between self-similar patterns of plastic wave front, fatigue crack kinetics, scale invariants characterizing the structure of materials and wide-range constitutive models [9,10,11].
Because of the difficulty in recording shock-wave loading parameters in LSP modes (a small spot size for measurements using VISAR technique), the methodological support for LSP relies on the registration of shock-wave pulse parameters during the plate impact tests of a massive target and data processing to determine the action invariants characterizing the formation of self-similar plastic fronts and the corresponding material structure during shock-wave pulse propagation in a target. The study of the material is conducted on samples from a massive target in VHCF tests, including the stages of fatigue crack initiation, propagation and sample separation, and is followed by a quantitative analysis of the fracture surface morphology by the method of interference profilometry. Additionally, scale invariants that characterize fatigue crack initiation and propagation are determined.
By comparing the shock-wave and fatigue loading data through action invariants and scale invariants that characterize shock-wave treatment modes and the staging of fatigue failure, an approach to optimizing laser treatment conditions to ensure the required service life of aircraft engine structures is discussed.
The objectives of the study are:
- Develop a concept for describing structural changes in terms of action invariants corresponding to universal power laws governing the formation of structured wave fronts and the kinetics of fatigue crack advance;
- Develop an experimental methodology for determining action invariants based on verified set-ups for implementing successive shock-wave and fatigue loading;
- Verify structure-sensitive parameters associated with characteristic scales determining the correlated behavior of defects during the formation of structured wave fronts, the size of the process zone during fatigue crack propagation, and the corresponding action invariants;
- Substantiation of the developed concept of describing the state of materials under successive shock-wave and fatigue loading using action invariants as applied to the optimization of LSP treatment.

ACTION INVARIANTS AND SELF-SIMILARITY OF PLASTIC WAVE FRONTS AND FATIGUE FAILURE

Introductory remarks

The application of the "action invariant" concept as a characteristic of shock-wave loading is natural in comparison with quasi-static and dynamic loading conditions due to the qualitatively different patterns of structural changes during the formation of plastic wave fronts. The self-similarity of plastic wave fronts (the Swegle & Grady fourth-power law [12,13]), which were established for a wide class of materials, determines the scale invariance of defects induced relaxation mechanism on plastic wave fronts. The ratio of velocity amplitude to wave front width determines the strain rates on the plastic wave front, the dependence of which on the stress amplitude is specified by the universal value of the power exponent, close to 4.

This type of self-similarity, known as similarity of the second kind [14], allows the introduction of self-similar "coordinates" and the reduction of plastic wave fronts for different load pulse amplitudes to a single dependence [15]. The integral characteristic of this similarity is the "action invariant", which reflects the property of anomalous "energy absorption" on the wave front scale [13]. The introduction of the "action invariant" as an integral parameter is associated with the defect induced "stored energy" mechanism as a physically alternative parameter to the "residual stress". The "residual stress" parameter is acceptable for quasi-static and dynamic loading (strain rates not exceeding 10^4 s^{-1}), but it is inapplicable to assessing shock-wave loading conditions. A justification for the self-similarity of plastic wave fronts, related to the collective properties of defects (Shear Bands), is proposed in [9,10], where the original experiments and the structural patterns after the plate impact test conducted under "recovery conditions" were analyzed. It was shown that the power-law universality of the plastic wave front corresponds to the self-similar structural patterns in the cross-section of the target in the direction the shock wave is travelling.

Comparing the self-similarity of plastic wave front (with power exponent close to 4) and the self-similarity of fatigue crack propagation with a similar power exponent in the Paris law is fundamentally significant. The power law exponent in the Paris law reflects the similarity of the physical mechanisms of Shear Bands in the "process zone" at the fatigue crack tip analogous for the power universality plastic wave front and defines the "critical conditions" for the correlated behavior of Shear Bands at the scale of the "process zone" preceding fatigue crack propagation. Taking into account the self-similarity of fatigue crack growth kinetics, the Paris law interpretation can be proposed using the action invariant, which is a consequence of the energy criterion for crack growth introduced in [16] and developed in terms of the Finite Fracture Mechanics (FFM) approaches and the Theory of Critical Distance (TCD) [17, 18].

Eliminating the artifacts characteristic of the LCP technique and associated with the small size of the "laser load spot" is crucial for determining optimal shock-wave loads for increasing fatigue life and FOD resistance. In this study, the problem is addressed by implementing the plane-wave loading on massive material targets and by recording the shock-wave pulse parameters by the Doppler interferometry to determine the "shock action invariant". Subsequent machining of standard fatigue test specimens from the target material enables interpreting the stages of fatigue failure using "fatigue action invariant" during accurate experiments. Comparing shock-wave and fatigue loading conditions in terms of "action invariants" makes it possible to propose an approach for optimization of shock-wave treatment to increase the fatigue life.

Action invariant of plastic wave fronts

For pressures of 1-10 GPa, typical for LSP treatment, the strength and viscosity effects become decisive in the formation of a shock wave profile [6,7]. A unique feature of large-amplitude wave profiles is the universality of the steady-state plastic wave front. The steady-state profile propagates without changing its shape, which is a consequence of a stable balance between competing processes: a nonlinear relationship between stress and strain and dissipative (viscous) properties of the medium caused by structural changes in the material. In [12], the dependence of the strain rate on the stress amplitude was experimentally studied, and a fourth-order dependence of the plastic strain rate on the stress amplitude was obtained.

$$\dot{\varepsilon}_b = D\sigma_b^4 \tag{1}$$

where D is the material parameter.

These data suggest a self-similarity of the shock wave in metals caused by the influence of the structure. The study of these mechanisms seems important for understanding the role of collective effects in the ensemble of defects responsible for plastic deformation of solids, as well as the influence of shock wave treatment on strength and fatigue properties.

Invariance as a four power law in a steady-state structured shock wave is observed for a fairly wide range of strain rates and shock compression amplitudes. In [13], the representation of this invariance in the form of a product of the dissipated energy and the time during which this energy is dissipated in the shock wave is considered. This product has a property with the dimension of action. It is noted that a steady-state "structured" shock wave propagates without changing its shape and is a consequence of a stable balance between competing processes of nonlinear stress-strain relationships, as well as the dispersion properties of the material during defect evolution.

This intriguing feature of stable shock waves in solids has also been observed for the reloaded regimes [15], which can be associated with multiple LSP. The fourth power law for a structured wave is closely related to the viscosity mechanisms of a solid and reflects the property of adiabatic invariance [13]. Dislocation dynamics and dispersion due to the scattering in

heterogeneous media have played a central role in explaining this invariance as a mechanism of shock wave failure via collective adiabatic shears.
The action invariant for structured wave front is written as

$$A_b = \int_0^{t_b} \delta E \cdot dt = \int_0^{\varepsilon_b} \delta E \cdot \frac{1}{\dot{\varepsilon}_b} d\varepsilon \tag{2}$$

where the increments of energy and time are shown in Fig. 1; $\varepsilon_b, \dot{\varepsilon}_b$ is the strain and strain rate at the plastic front; δE is the dissipation; σ(ε) is the dependence of dynamic stress on strain in the current position of the wave. The term σ_υ represents the "dissipative stress", which determines the amplitude of the dynamic stress exceeding the stress in the Equation of State (EoS) [13].

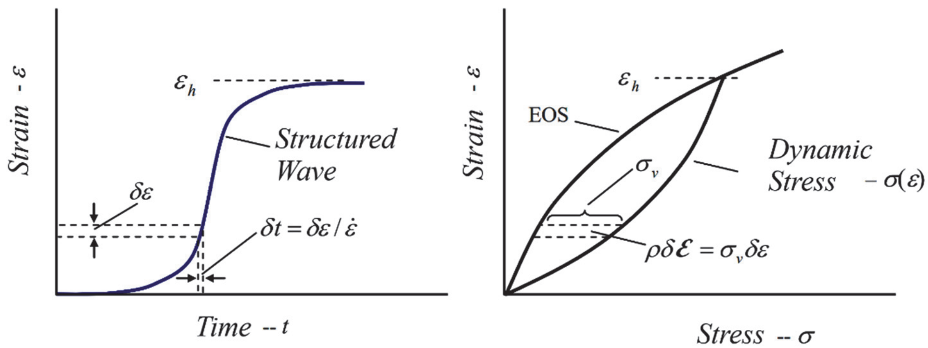


Figure 1: Illustration of stress dependence in a structured wave [13].

Eqn.2 determines the energy part performed by the shock wave and accumulated in the stress fields of defects (stored energy), which is not presented in the Equation of State (EoS) of the material. Dissipative mechanisms include dislocation motion, twinning, slip bands, which determine the adiabatic shear scenario of relaxation. A common feature of this mechanism is the dissipation of the kinetic energy of the lattice at acoustic frequencies characteristic of multiscale nucleation and growth of defects.
Another aspect should be noted concerning the experimental data on the four power dependence between the strain rate and stress in a structured shock wave. Power relations in physics reflect the universality of the mechanisms that determine the relationship between the dynamics of internal (structural) and observable variables under shock-wave and fatigue loading [14].
In this case, power relations assume that there is no determining scale of stresses in the range of power-law behavior. In many applications, the power-law behavior manifests itself in a limited range of loading parameters. Such behavior is usually associated with the existence of the so-called self-similar intermediate asymptotic or self-similarity of the second kind for structural variables, and it is observed, for example, in the theory of critical phenomena, during the development of detonation, hydrodynamic turbulence, as well as during fatigue failure [19, 20].
The integral of action, as the energy accumulated in the stress fields of defects generated by a shock wave, can be used as a parameter characterizing the state of the material structure after shock-wave loading. The relationship between the action invariants in plastic wave fronts and fatigue crack advance can be used as the methodology for optimizing shock-wave treatment to ensure the subsequent fatigue life of materials.

Action Invariant of fatigue crack advance

Self-similar aspects of fatigue failure and its stages are the subject of persistent interest [21,22]. In accordance with the signs of self-similarity, the power law for High Cycle Fatigue (the Paris law) is considered as a consequence of nonlinearity of defect dynamics, subordinating the behavior of the experimentally observed variables (stress intensity factor, crack length). Self-similar regularities phenomenologically corresponding to the Paris law are considered for HCF and VHCF generalized for the case of small cracks [23].
The crack advance kinetics in the Paris power law reads

$$\frac{da}{dN} = C\left(\Delta K\right)^{m} \tag{3}$$

where $\Delta K = K_{\max} - K_{\min}$ ($K_{\max}$ and $K_{\min}$ are the maximum and minimum stress intensities in the fatigue cycle, respectively); *C* and *m* are the experimentally determined constants. The presence of intermediate asymptotic suggests the existence of "material constants" that determine the characteristic scales of interaction when the observed variables (da/dN and ΔK) obey the nonlinear kinetics of damage in the process zone with the characteristic scale *L*. These mechanisms are reflected in the exponent *m* and the intermediate-asymptotic nature of damage localization, ensuring the development of a fatigue crack.

Phenomenologically, the role of scales is taken into account in the approaches of the Finite Fracture Mechanics (FFM) and the Theory of Critical Distance (TCD) with estimation of the process zone length L as [17, 18]

$$L = \frac{1}{\pi}\left(\frac{K_c}{\sigma_c}\right)^2 \tag{4}$$

where K_c is the critical value of the stress intensity factor; σ_c is the strength limit.

The length L can also be determined from energy balance using the FFM and TCD concepts of the finite increment length during crack propagation [17].

$$\int_0^L K^2 da = K_c^2 \cdot L \tag{5}$$

Eqn.5 characterizes the "viscous fracture" work during crack propagation, and it is used further to determine the "action invariant" during fatigue crack propagation. The finite crack length increment in FFM was introduced as a conceptual theoretical basis, suggesting that crack propagation occurs in intermittent or branched dynamics, and its size is determined by the microstructure and deformation behavior of the material [18]. For fatigue cracks, L corresponds to the process zone and is associated with the microstructural parameter (grain size) and damage localization. The size of the process zone is also associated with the concept of the critical distance and determines the number of damage localization zones required to form a destruction zone (daughter crack), the formation of which ensures the propagation of the main crack [24].

Generalization of FFM to fatigue problems assumes that fatigue damage depends on the stress field distribution in the vicinity of the fatigue crack, and fatigue damage can correctly be estimated by the entire stress field [25].

According to the TCD, the fatigue limit condition can be formulated in the terms of the effective stress $\Delta\sigma_{eff}$, which depends on the maximum principal stress distribution ahead of the crack tip and equals the material plain fatigue limit, $\Delta\sigma_0$

$$\Delta\sigma_{eff} = \Delta\sigma_0$$

The critical distance can be calculated as follows [25]:

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_0}\right)^2, \tag{6}$$

where ΔK_{th} is the range of the threshold value of the stress intensity factor, and $\Delta\sigma_0$ is the plain fatigue limit (both values are determined under the same load ratio, R). Similar to (5), L represents a characteristic length of a material, which can vary depending on its structure and load ratios.

The FFM criterion can be reformulated as follows:

$$\Pi_f = \int_0^L \left(\Delta K_{eff}\right)^2 da = \left(\Delta K_{th}\right)^2 \cdot L \tag{7}$$

Taking into account the dimension of Π_f, the energy in the process zone can be estimated as

$$E_f = \Pi_f^{1/2} \cdot L^2 \tag{8}$$

and, consequently, the fatigue action invariant A_f reads:

$$A_f = E_f \cdot T_f \tag{9}$$

where T_f is the characteristic time of fatigue crack advance related to the time of critical damage localization in the process zone. The introduction of the fatigue action invariant A_f is strongly related to the signs of power self-similarity signs in the Paris laws (3), as mentioned for the shock wave action invariant A_h and the power universality of plastic wave fronts. Both these invariants are used for estimation of the state of material subject to the consecutive shock wave loading to provide fatigue resistance.

Action invariants as optimization parameters for the shock-wave treatment to ensure fatigue life

The introduced action invariants correspond to the self-similar laws of formation of structured plastic wave fronts (the Swegle-Grady power universality) and the fatigue cracks kinetics (the Paris law) and are used as optimization relations for determining the parameters of shock-wave loading (amplitude, pulse duration), ensuring the maximum fatigue life under subsequent HCF and VHCF loading.
The amplitude σ_h and the pulse duration t_h in the "shock-wave action invariant" $A_h(\sigma_h, \tau_h)$ and, accordingly, the threshold value of the stress intensity factor ΔK_{th} and the plain fatigue limit $\Delta\sigma_0$ in the "fatigue action invariant" $A_f\left(\Delta K_{th}, \Delta\sigma_0\right)$ are utilized as optimization parameters. The optimization problem is formulated in terms of time T_f as a minimax problem of experimental determination of a limited set of variables $\left\{\sigma_h, t_h, \Delta K_{th}, \Delta\sigma_0\right\}$.

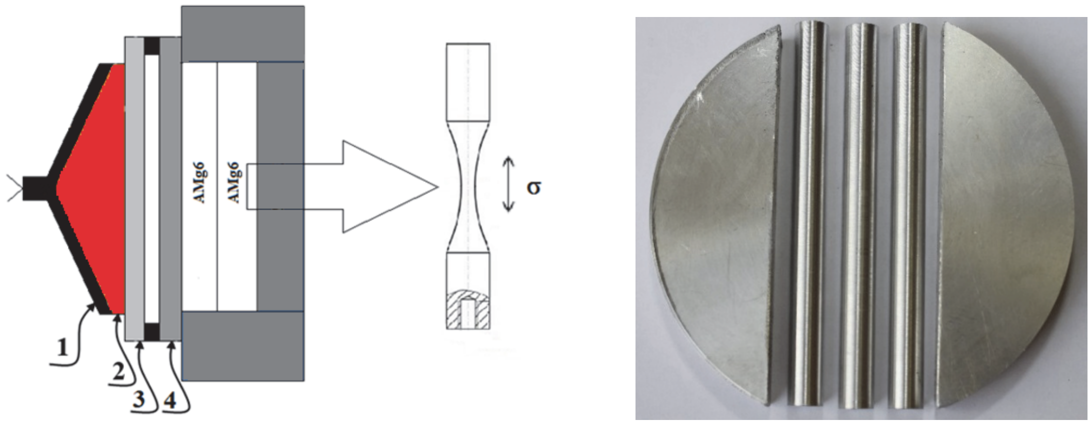
$$T_f\left(Mini - Max\right) \sim \Xi\left\{\sigma_h, t_h, \Delta K_{th}, \Delta\sigma_0\right\} \tag{10}$$

where $\Xi\left\{\sigma_h, t_h, \Delta K_{th}, \Delta\sigma_0\right\}$ is the objective functional.

Optimal shock-wave treatment corresponds to the maximum crack propagation time T_f 8 determined by the properties of the material subjected to shock-wave loading. All material parameters are calculated using the obtained experimental data. The constructed dependencies are applied to solve an optimization problem to ensure the maximum fatigue life. The optimization criterion, which is based on a comparison of action invariants, reflects the fundamental patterns determined by the self-similar structure of plastic wave fronts (fourth-order in the Swegle-Grady relations) and the self-similarity of fatigue crack propagation kinetics (the Paris law).

Experimental study of fatigue resistance of alloy subjected to shock-wave loading

Following the methodology developed by the authors [26] in the study of the material tolerance under consecutive dynamic and fatigue loads specified for the Foreign Object Damage (FOD) problem of aircraft engine alloys, the recent research is actualized on a generalization of the approach for the shock-wave preloading of alloys corresponding to the LSP treatment. The methodological aspect of the research is associated with the shock-wave processing of massive targets with the registration of shock-wave parameters by the Doppler interferometry method [27]. The subsequent machining of the samples from the target material was used for the study of fatigue staging and the quantitative analysis of fracture surface patterns. An AMg6 aluminum alloy target plate, 15 mm thick, was subjected to preliminary loading via impact from a 4 mm-thick, 120 mm-diameter aluminum plate projectile. The projectile was accelerated to a velocity of approximately 1400 m/s using a plane wave generator (Fig. 2a). The impact was conducted under recovery conditions, which permitted the subsequent extraction of specimens from the preloaded target for very high cycle fatigue (VHCF) testing at room temperature.



a) b)

Figure 2: a) Schematic diagram of the experimental set-up for shock-wave loading of AMg6 alloy targets: 1- explosive conical lens; 2- paraffin insert; 3- steel shock absorber plate; 4- aluminum projectile [24]. b) Photograph of the target (a) with cutout samples

The selection of the impact plate thickness for the specified impact velocity was governed by two criteria: the establishment of a shock-compressed state and the maximization of the compression pulse amplitude through the target's entire thickness. To mitigate lateral and rear unloading waves and thus to avoid additional sample deformation, the target was interference-fitted into a confining steel ring with a diameter of 200 mm. The configuration of the experimental setup allowed implementing impact deformation in both targets only in the direction perpendicular to their main plane. The implemented approach allows obtaining the macroscopic samples pre-loaded with a controlled pulse under plane-wave loading conditions, which eliminates a number of artifacts associated with the study of the state of the material subjected to LSP, such as the size of a small spot on the target surface, preventing the implementation of modes close to the plane-wave ones. It also makes it possible to get reliable information from VISAR data, as well as an indirect estimate of the parameters of the shock-wave in the target.
For VHCF testing, specially shaped samples, machined from the preloaded plates, were used (Fig. 2b).
Fatigue tests were performed using the Shimadzu USF-2000 resonant testing machine at stress levels from 90 to 162 MPa and a symmetrical cycle R = -1 with a frequency of 20 kHz. During the experiment, the samples were cooled with compressed air. A frequency deviation of 0.5 kHz was associated with a change in the mechanical impedance of the sample during the damage. The failure precursor was associated with a crack with a characteristic size of ~2 mm. Fatigue tests were conducted at stress amplitudes of 100–200 MPa. The level of applied stresses made it possible to study the fatigue life up to the values corresponding to 10^{10} loading cycles. The initiation of an internal fatigue crack was determined using the amplitude-frequency analysis (Fig. 3) of the changes in the effective elastic properties of the material [23, 27].

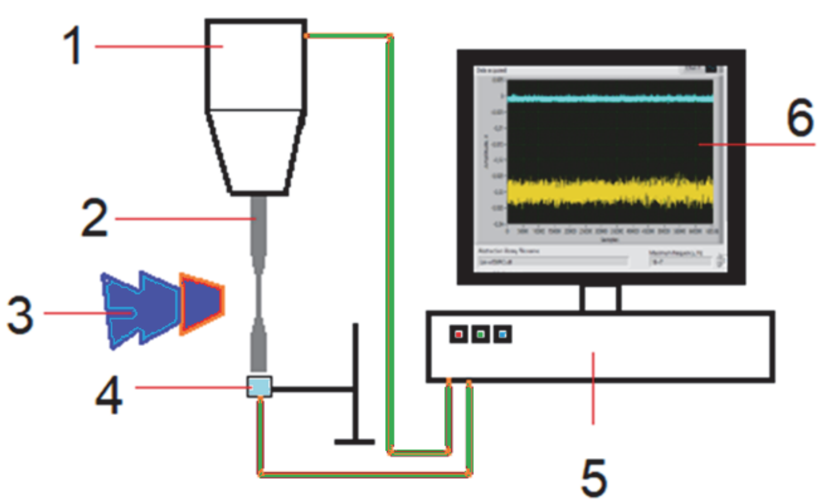


Figure 3: Experimental setup for VHCF testing: 1 - waveguide, 2 – specimen, 3 - cooling system, 4 - displacement sensor, 5 - control system and analog-to-digital converter, 6 - analysis software [23].

Ultrasonic testing was used in combination with a highly sensitive induction sensor and an analog-to-digital converter system that measured the amplitudes and frequencies of the free end surface vibrations of the sample to determine the values of the first, second and third harmonics and their amplitudes. The extent of structural changes and the progression of damage were evaluated using the parameter β, as defined within a nonlinear elasticity framework [23].

$$\sigma = E\left(\varepsilon - \frac{1}{2}\beta\varepsilon^2\right) \quad . \tag{13}$$

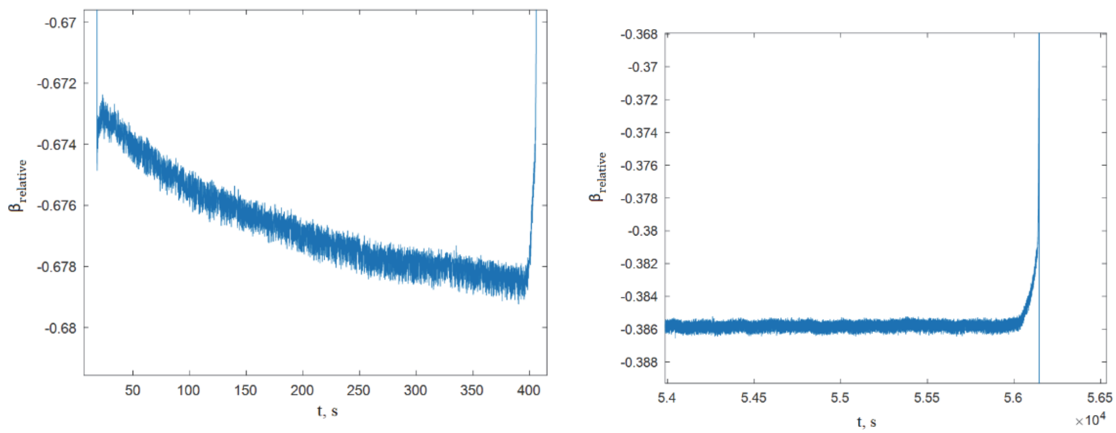


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Figure 4: Relative nonlinearity coefficient $\beta_{relative}=\beta/\beta_0$ (β_0 corresponds to the initial material state) at the precritical stage of cyclic loading of the sample subjected to preliminary shock: (a) σ = 150 MPa, N_f = 3.10×10^7; (b) σ = 110 MPa, N_f = 1.04 ×10^9) [27].

The analysis of the oscillations of the free end surface of the specimen made it possible to reveal the difference in the kinetics of fatigue crack initiation caused by the damage localization. For example, the fatigue test of the specimen at a load amplitude of 150 MPa and the critical number of cycles 3.10×10^7 revealed a monotonic trend in the change of the nonlinearity coefficient (Fig. 4a). At the critical stage associated with crack initiation, the value of the $\beta_{relative}$ coefficient increases sharply, by approximately an order of magnitude. For the specimen tested for fatigue at a small loading amplitude of 110 MPa and the critical number of cycles 1.04×10^9, the nonlinearity coefficient reaches a constant value and remains unchanged for 99% of the life time (Fig. 4b). During the final experimental stage, the nonlinearity coefficient exhibits avalanche-like growth. This sharp increase is attributed to the long-term accumulation of defects until a critical point is reached. Further increase in the number or size of defects to a certain critical value leads to the initiation of an internal fatigue crack, which very quickly spreads in the material, causing its failure.

SCALING INVARIANCE OF THE FRACTURE SURFACE UNDER CONSECUTIVE SHOCK-WAVE AND FATIGUE LOADS

The Critical Distance as the scale invariant

The power law reflects the collective behavior of defects in the process zone, ensuring the staging of damage-failure transition with characteristic manifestation of self-similarity [22]. The methodology of self-similar intermediate asymptotic [14, 22, 23] was applied to the analysis of the Paris law to describe the fatigue crack advance with the power law kinetics.

Power laws for the kinetics of small cracks and Paris cracks are discussed in [19,28] by analyzing the scale invariants of the roughness profiles, which characterize the zones of initiation and propagation of fatigue cracks in the HCF and VHCF modes using the replica technique [28] and high-resolution profilometry [26]

.The scaling invariant in terms of the Hurst exponent H was estimated by averaging the difference in roughness heights $z(x)$ on the Process Zone surface according to the formula [22]:

$$C(r)=\left\langle (z(x+r)-z(x))^2 \right\rangle_x^{(1/2)} \propto r^H \tag{14}$$

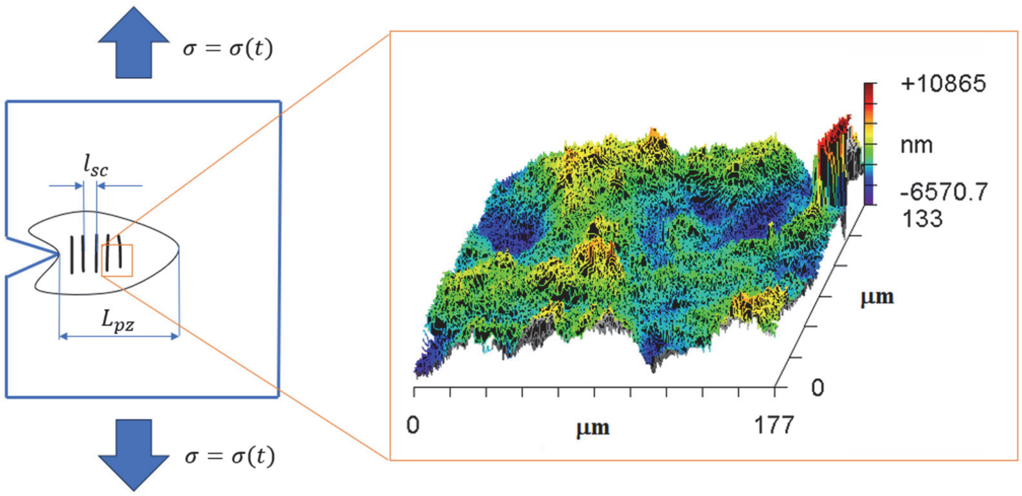


Figure 7: Schematic image of the process zone at the crack tip and typical image of the oblique surface roughness [28].

The log-log plot $\log_2 C(r) \sim \log_2(r)$ allowed the estimation of the roughness exponent as a spatial invariant in corresponding range of scales $[l_{sc}, L_{pz}]$. The value of the lower boundary of the linear section of the function C(r) was taken as the value of the critical scale l_{sc}, i.e. the minimum spatial scale in the process zone, at which the scale-invariant roughness pattern manifests itself as a defect induced structural scale. The value of the upper boundary L_{pz} was taken as the length associated with the maximum area of correlated roughness behavior. The length l_{sc} determines the so-called cross-over point separating the long-range correlation corresponding to the transition from small to the Paris crack advance. The length L_{pz} is associated with the Critical Distance *L* [24].

For a wide class of materials and different crack growth rates under HCF, the exponent m is close to the values of m~4. However, the traditional formulation of the Paris law becomes inapplicable for small cracks or under low stress conditions, when the microstructure and damage exert a significant influence on the crack kinetics. To describe the crack kinetics for sizes smaller than the size of the "Paris cracks", a phenomenological relationship was proposed in [22]. The self-similar patterns of fracture surface in the conditions of HCF and VHCF were studied using the similarity theory and dimensional analysis [23, 29, 30]. The dependence of the crack growth rate da/dN was determined by the following list of parameters:

$$\frac{da}{dN}=F(\Delta K, G, l_{sc}, L_{pz}) \tag{15}$$

where ΔK is the range of the stress intensity factor; *G* is the shear modulus; l_{sc} is the minimal spatial scale in the process zone at which the scale-invariant patterns of the fracture surface roughness to manifest the self-similarity; L_{pz} is the scale related to fracture process zone. Using the Π -theorem, Eqn. (15) can be represented as

$$\frac{da}{dN}=\Phi\left(\frac{\Delta K}{G\sqrt{l_{sc}}}, \frac{L_{pz}}{l_{sc}}\right) \tag{16}$$

An estimate of the values $\Delta K / E\sqrt{l_{sc}} << 1$ and $L_{pz} / l_{sc} >> 1$ allows the assumption on the intermediate-asymptotic nature of the crack kinetics and represents (8) in the form:

$$\frac{da}{dN}=l_{sc}\left(\frac{\Delta K}{G\sqrt{l_{sc}}}\right)^{\alpha}\left(\frac{L_{pz}}{l_{sc}}\right)^{\beta} \tag{17}$$

where α and β are the power exponents reflecting the intermediate-asymptotic nature of crack kinetics as a function of dimensionless variables $\Delta K / \left(G\sqrt{l_{sc}}\right), L_{pz}/l_{sc}$. The parameter $\Delta K_{sc} = \Delta K \left(L_{pz}/l_{sc}\right)^{\beta/\alpha}$ is introduced. This allows us to write Eqn. (17) in a form similar to the Paris law

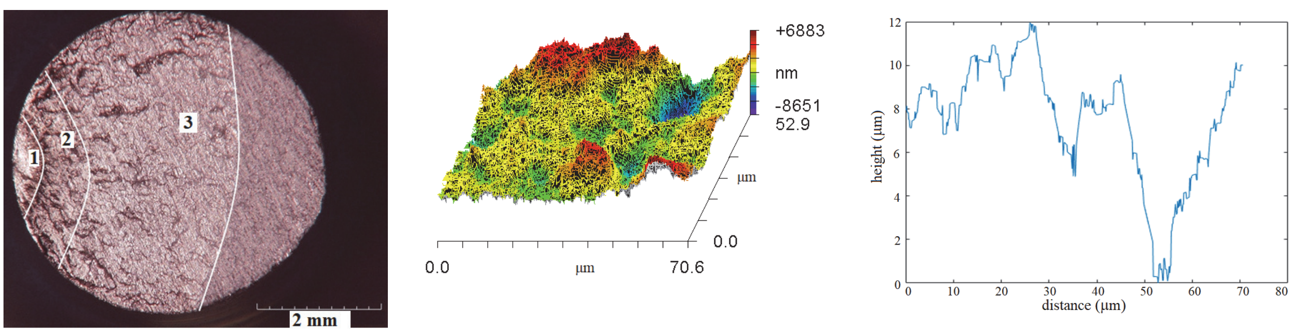
$$\frac{da}{dN} = l_{sc}\left(\frac{\Delta K_{sc}}{G\sqrt{l_{sc}}}\right)^{\alpha} \tag{18}$$

Kinetic Eqn. (18) is suitable for modeling the growth of both small and large cracks, whose behavior is governed by the structural parameters and scaling exponents. These power-law exponents are intrinsically linked to the dominant mechanisms responsible for free energy release within the process zone and the value of the fatigue action invariant $A_f\left(\Delta K_{th}, \Delta\sigma_0\right)$. Under a "plastic scenario," the formation of critical number of Slip Bands occurs on the L_{pz} scale associated with the threshold ΔK_{th}. Reaching the correlated behavior of SB dictates the exponent in the Paris law.

Quantitative analysis of the fracture surface roughness

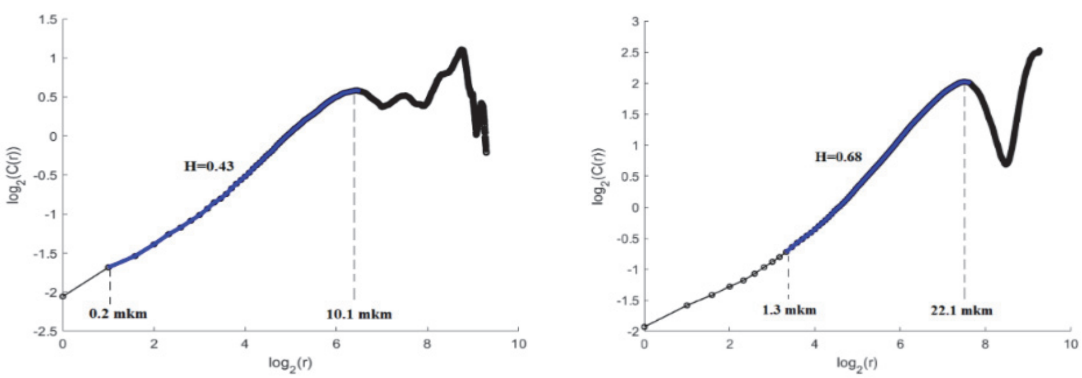
The aim of this study is to define the characteristic structural lengths responsible for the correlated behavior of defects and the staging of fatigue damage-failure transition in the consecutive shock wave and fatigue loads.

The quantitative fractographic study of the fracture surface of the samples subjected to consequent shock-wave and fatigue loads was carried out using the New-View 5000 interferometer (at x2000 magnification, Fig. 8).



a b c

Figure 8: (a)Typical images of fatigue failure zones in AMg6 alloy (σ = 121 MPa, $N_f = 7.29\times10^7$): 1 – fatigue crack initiation zone, 2 – fatigue crack growth zone, 3 – fatigue crack propagation zone according to the Paris kinetics). (b) typical three-dimensional profile of zone 2; (c) typical one-dimensional profile of zone 2.



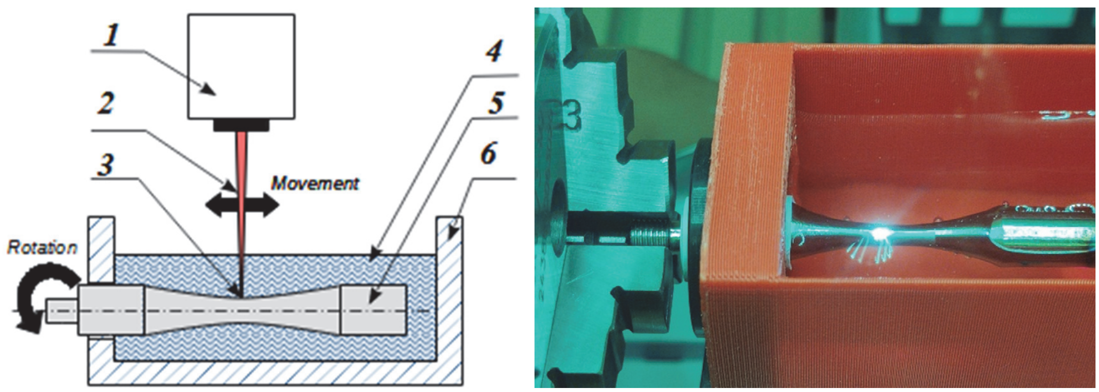
a b

Figure 9: Typical form of the dependence of $\log_2 C(r)$ on $\log_2(r)$ for samples: a) – with preliminary loading; (b) – without preliminary loading [24].

The comparative analysis of scale invariants, which is based on quantitative profilometry data, revealed a decrease in the Hurst exponent (H=0.34-0.58 in the structural scale range of 0.3-30.7 µm) for pre-loaded samples and an increase in the exponent (H=0.58-0.76 in the structural scale range of 1.3-34.7 µm) for unloaded samples. The structure of the material loaded at high strain rates is characterized by intense fragmentation during the formation of dislocation ensembles, which initiate a multi-scale defect growth during subsequent fatigue loading. The initial fragmented structure complicates the formation of an ordered defect system during fatigue loading, which is apparently the reason for the decrease in the Hurst exponent and a less pronounced ability of the material to localize damage in the form of persistent slip bands at the fatigue crack tip.

ON PECULIARITIES OF LASER SURFACE TREATMENT OF MATERIALS

Along with the study of shock-wave preloading, it is of interest to analyze the influence of laser surface treatment on the fatigue resistance [31].
The study of the cylindrical working part of the VT6 alloy samples was conducted using a laser set-up MiniMarker 2 – 30A4 RA and a high-precision rotator through a 2 mm thick layer of water. This initiates the shock-wave pulse in the material due to the formation of plasma in a thin surface layer of the material under laser radiation. The installation and treatment of samples are shown in Fig. 10.



a b

Figure 10: Schematic diagram of the experimental setup based on the Minimarker 2 precision laser marking system (PLMS) and the process of treatment of cylindrical samples: 1 – laser head with lens; 2 – laser beam; 3 – laser beam focus point (on the sample surface); 4 – water, 2 mm layer; 5 – sample.

The setup includes an ytterbium nanosecond pulsed fiber laser with a wavelength of 1064 nm manufactured by IPG Photonics. The irradiation parameters are: the pulse duration of 200 ns, the pulse energy of 1 mJ, diameter of the spot of the laser beam focused on the surface of ~ 30 μm. To establish the effect of laser treatment on the damage patterns of the titanium alloy VT6 under VHCF loads, the working parts of the cylindrical samples were processed in the scanning mode, and the laser beam moved along the surface of the sample "track by track" so that the impact zones (spots) touched but did not overlap. After each pass of the line, the sample was rotated around its axis by a given angle so that the lines touched but did not overlap, thereby processing the entire surface in the working area of the sample.
Since the surface quality plays an important role in VHCF, the samples with different surface quality were tested to separate the effects of "roughness" and "laser treatment": Ra = 0.25 ± 0.05 μm and Ra = 0.15 ± 0.05 μm in the initial state and after laser processing. After laser treatment, the surface roughness of the samples was on average Ra = 1.30 ± 0.10 μm. To obtain the specified level of roughness (Ra = 0.15 μm and Ra = 0.25 μm), the samples processed on the Minimarker 2 SPLM were subjected to fine grinding (from P600 to P2000) and polishing with diamond paste with an abrasive size of 1-2 μm. Grinding was used to remove the relief of craters, but with the preservation of a thin hardened layer. The fatigue testing of titanium alloy samples was carried out in the symmetrical loading mode "tension-compression" at a frequency of 20 kHz on a Shimadzu USF-2000 ultrasonic fatigue machine. The test results are presented in Tab. 1.
The obtained results (Tab. 1) show that the treatment performed according to the proposed mode on the SPLM "Minimarker 2" set-up leads to an increase in operational characteristics by 9-10% during VHCF. The method used made it possible to separate the influence of roughness and laser treatment on the fatigue properties of the alloy. It is shown that the laser surface processing with subsequent grinding is a promising means of increasing the fatigue life of materials for aircraft engine structures.

Treatment	Surface quality Ra, μm			
	0,25 ± 0,05		0,15 ± 0,05	
	σ, MPa	ΔN, cycles	σ, MPa	ΔN, cycles
Original state	460	$5.47 \cdot 10^8$	460	$7.73 \cdot 10^8$
Laser surface treatment	475	$2.22 \cdot 10^9$	490	$3.48 \cdot 10^7$

Table 1: Results of the fatigue tests on the initial and surface laser-treated samples of Ti–6Al–4V alloy with different surface qualities.

DISCUSSION OF RESULTS

The shock-wave treatment of materials, like LSP, involves establishing a link between the deformation mechanisms during the propagation of an elastic-plastic wave and the structural changes in the material. This makes it possible to provide the required properties, in particular, the fatigue life under operating conditions, including, for example, the situations of an accidental impact of fan blades with foreign object damage (FOD), which are common for applications. In comparison with dynamic loading, the shock-wave action is characterized by the fundamental differences in structural changes associated with the patterns of formation of elastic-plastic fronts and their effect on the fatigue properties of materials and structures. A fundamental feature is the self-similarity of the plastic wave front, caused by the correlation of structural changes due to numerous Shear Bands at the scale of the wave front. This is an important feature of the non-local interactions in defect ensembles that cannot be reflected in traditional criteria used in the case of quasi-static and dynamic effects and based, for example, on the assessment of residual stress levels. The non-local properties of damage in the materials under the impact of a shock-wave pulse also manifest themselves under subsequent fatigue loadings, influencing both the patterns of initiation and growth of fatigue cracks. Given the short duration of the shock-wave pulse determined by the plastic front width, for assessing the fatigue life, it is of fundamental importance to establish the relationship between the structure of the material formed under shock-wave loading and the stages of initiation of small fatigue cracks, their growth to the scale of Paris cracks and propagation. This implies taking into account the properties of the material with defects formed during the propagation of the plastic wave.

These features of structural changes during propagation of an elastic-plastic pulse and their influence on the patterns of initiation and propagation of fatigue cracks should be reflected in non-local criteria based on the invariant characteristics of the material under shock-wave and fatigue loads. In this case, it is necessary to determine these invariant characteristics based on the methodologically substantiated statements that allow recording the parameters of shock-wave and subsequent fatigue loading in comparison with the results of structural studies. Methodological statements for shock-wave loading suggest implementation of plane-wave loading of samples and recording by interferometry methods to eliminate the artifacts of measurements of shock-wave pulse parameters under LSP conditions with a small value of the impact spot and uncertainty in the magnitude of the loading pulse under ablation conditions, formation of a plasma torch initiating a compression shock pulse. At the same time, the model experiments on plane-wave loading are basic for determining the invariant characteristics, which provide the relationship between the parameters of the shock-wave pulse and the changes in the structure in terms of the "action invariant", the value of which is the parameter of optimization of the structure for assessing the fatigue life. Known cases of violation of self-similarity of structured wave fronts (for example, for vanadium) correspond to special plasticity mechanisms, which can also be interpreted in terms of action invariants with exponents that take these mechanisms into account.

The parameters of non-locality, structural scales characterizing the development of fatigue failure, are determined based on the quantitative fractography data and allow for the assessment of the "action invariant" for a quantitative description of the kinetics of fatigue crack growth, taking into account changes in the structure formed under shock-wave loading.

The specific features of shock-wave loading and the qualitatively different mechanisms of structure formation and damage in compression and rarefaction waves suggest the reflection of these mechanisms in wide-range models linking the mechanisms of structural relaxation, multiscale kinetics of defects with the self-similar regularities of formation and propagation of shock-wave pulses. In modeling the dynamics of elastic-plastic fronts, the connection with the mechanisms of structural relaxation and the staging of fatigue damage-failure transition suggests the development of special algorithms and software for performing calculations applicable to the real processes of high-energy and shock-wave treatment.

The results of the research allow us to draw the following conclusions on the substantiation of the methodological aspects of fatigue life assessment during the preliminary shock-wave treatment of materials:

1. The formation of the structure during shock-wave treatment is determined by the self-similar regularities of the elastic-plastic wave profile, which can be identified by introducing an "action invariant" that links the shock wave parameters (amplitude, wave front width) with the dissipative properties of the material caused by defects (stored energy).

2. The introduction of an "action invariant" enables describing the kinetics of fatigue crack development in the materials subjected to preliminary shock-wave loads, including the initiation and development of small cracks, the Paris cracks.

3. The quantitative fractographic analysis using the interferometry data for fracture surfaces made it possible to identify structural scales for determining the "action invariants" that determine the kinetics of fatigue crack in materials after shock-wave treatment.

4. Methodological principles for studying the behavior of materials under consecutive shock-wave and fatigue loads have been developed, allowing for optimization of shock-wave treatment to ensure fatigue life.

5. With regard to LSP of materials in aircraft engine construction, it is necessary to implement a full experimental and research cycle that includes stages such as

- conducting experiments with plane-wave loading of targets to determine the values of the "action invariant" of structured wave fronts at different wave pulse amplitudes and obtaining standard samples for fatigue testing;
- conducting fatigue tests on standard samples in combination with structural analysis methods to determine the "action invariants" related to the kinetics of fatigue crack;
- substantiation of models to describe shock-wave fronts in the characteristic ranges of LSP treatment, followed by an experimental study of the effect of this treatment for model LSP settings.

6. Development of optimization algorithms using "action invariants".

The listed items represent the directions of detailed research into the characteristics of shock-wave loading and its relationship with fatigue life assessment using the concept of action invariants, including LSP treatment

Author contribution

Author contributions are following. Conceptualization and supervision: O.B.N. (Oleg B. Naimark); methodology A.N.B (Aleksandr N. Balakhnin), M.V.B. (Mikhail V. Bannikov), V.A.O. (Vladimir A. Oborin), S.V.U. (Sergey V. Uvarov), A.D.Yu. (Aleksandra D. Yurina) ; writing-original draft preparation: O.B.N., V.A.O., M.V.B.

Acknowledgements

The work was carried out as part of a major scientific project funded by the Ministry of Science and Higher Education of the Russian Federation (Agreement No. 075-15-2024-535 dated 23 April 2024).

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