A Novel Sparse Fourier Orthogonal Coding-Based DOA Estimation Technique Using a Hermitian Propagator and Covariance Projection

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ABSTRACT

This paper presents a robust method for Direction of Arrival (DOA) estimation using Sparse Fourier Orthogonal Coding (FOC). High-precision DOA estimation in radar, sonar, and wireless systems often suffers from noise, interference, and limited resolution. To overcome these issues, this study proposes a framework that integrates Sparse FOC, Hermitian Propagator, and Sparse Toeplitz Covariance Matrix Projection, forming the Sparse Hermitian Propagator for DOA Estimation (SHP-DE). The proposed method improves estimation by decomposing received signals into orthogonal components, improving feature extraction and information diversity. The Hermitian Propagator eliminates the need for eigenvalue or singular value decomposition, reducing computational complexity without sacrificing accuracy. The Toeplitz Covariance Projection further refines the covariance structure, enhancing noise suppression and estimation stability. Simulation results demonstrate that SHP-DE achieves superior resolution and robustness, outperforming traditional algorithms, such as MUSIC and ESPRIT, in various noisy and closely spaced source scenarios. The ability of the proposed method to maintain performance under challenging conditions marks a significant step toward practical real-time DOA estimation. Thus, SHP-DE is well-suited for applications demanding reliable and accurate localization of signal sources in complex environments.

Keywords-Direction of Arrival (DOA) estimation; sparse Fourier Orthogonal Coding (FOC); Sparse Hermitian Propagator (SHP-DE); Hermitian propagator; sparse Toeplitz covariance matrix projection; high-resolution estimation; noise suppression; computational efficiency; MUSIC; ESPRIT; signal processing

I. INTRODUCTION

Direction Of Arrival (DOA) estimation is essential in many processing applications, such as sonar systems for underwater navigation, wireless communication networks for beamforming, and radar systems for target tracking. Accurate estimation of DOA offers improved signal reception, efficient interference mitigation, and improved spatial filtering in these domains [1]. High-precision estimation is still difficult to achieve in real-world situations, especially when noise, multipath propagation, and closely spaced signal sources are present [2].

For DOA estimation, conventional methods such as ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) and MUSIC (Multiple Signal Classification) have been widely used [3, 4]. Although these techniques achieve good results in best-case scenarios, their applicability in real-world situations is limited because they perform noticeably worse in noisy environments or with low Signal-to-Noise Ratios (SNRs) [5, 6]. The proposed Sparse Hermitian Propagator for DOA Estimation (SHP-DE) is a novel technique that utilizes sparsity and orthogonality principles to achieve high estimation accuracy and robustness to overcome these limitations [7, 8]. To enhance performance in demanding signal environments, the SHP-DE approach combines Sparse Fourier Orthogonal Coding (SFOC), a Hermitian propagator framework, and sparse Toeplitz covariance matrix projection [9, 10]. SHP-DE improves resolution, reduces noise, and allows for more effective signal representation by utilizing sparsity [11]. Additionally, the orthogonality-based design provides a computationally efficient solution by avoiding the need for explicit subspace decomposition. Consequently, SHP-DE outperforms traditional algorithms, particularly in complex scenarios that involve a lot of noise [12].

The novelty of the proposed SHP-DE method lies in the strategic integration of three key innovations: (i) SFOC for enhanced signal representation through orthogonal decomposition, (ii) a new Hermitian matrix propagator formulation that eliminates the need for Eigenvalue Decomposition (EVD) or Singular Value Decomposition (SVD), and (iii) the projection of the covariance matrix onto a sparse Toeplitz structure for noise suppression and subspace refinement.

A. Related Works

Most previous studies on DOA estimation focused on machine learning-driven approaches, subspace-based methods, and sparse signal reconstruction techniques. Among classical methods, MUSIC and ESPRIT are well-known subspace algorithms to estimate DOA using the concepts of eigenvalue decomposition and rotational invariance, respectively [13, 14]. The accuracy of these methods declines significantly when there is strong noise or sources that are closely spaced and correlated, but they perform well when the SNR is high [15, 16].

To overcome these restrictions, scientists have looked into sparse representation-based methods and Compressed Sensing (CS), which take advantage of the natural sparsity of source locations to improve resolution and robustness. Methods such as Bayesian inference and L1-minimization have been used to increase the accuracy of the estimation [17, 18]. However, the high computational complexity of these approaches is a major obstacle to real-time applications. To enhance DOA performance, recent studies have introduced hybrid frameworks that combine subspace estimation with deep learning models, going beyond sparsity-driven approaches [19]. Deep reinforcement learning algorithms and neural networks have proven to be more resilient in noisy and dynamic environments. Moreover, structured covariance estimation methods have been developed to improve noise resilience, including Toeplitz-based methods.

To improve estimation accuracy and computational efficiency, the proposed SHP-DE approach builds on these foundations by combining SFOC, the Hermitian propagator, and sparse Toeplitz covariance matrix projection [20, 21]. SHP-DE successfully overcomes the drawbacks of conventional methods and provides a reliable solution for DOA estimation in complex signal environments by utilizing both sparsity and orthogonality [21].

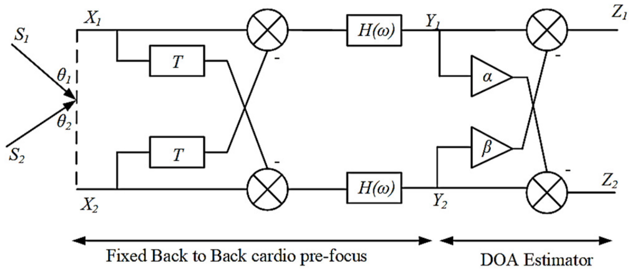

Fig. 1. Block diagram of the proposed sparse Hermitian propagator-based DOA estimation system.

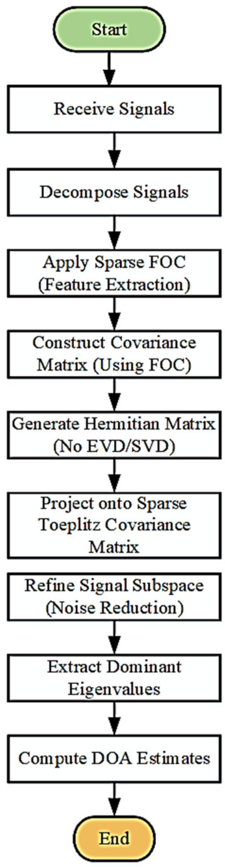

Fig. 2. Signal processing flow of the SHP-DE algorithm from input acquisition to DOA output.

II. PROPOSED METHOD

SHP-DE is intended to precisely compute the DOA of incoming signals in difficult and noisy situations. This strategy is especially designed to address the drawbacks of traditional estimation techniques, which underperform in practical settings. Figure 1 shows the block diagram for the suggested DOA estimation system. Even in noisy and correlated environments, the system is built to estimate the angles of the incoming signals with improved accuracy and robustness.

Figure 2 displays the sequential steps of the SHP-DE algorithm. Sparse FOC is used for feature decomposition after signal acquisition. Following the construction of a structured covariance matrix, a Hermitian matrix is produced without the need for traditional EVD or SVD. The extraction of dominant eigenvalues and precise DOA estimation are the results of further refinement using sparse Toeplitz covariance projection, which improves sparsity and lowers noise. SHP-DE integrates three fundamental components: SFOC, a Hermitian propagator-based spatial spectrum framework, and sparse Toeplitz covariance matrix projection. This combination reduces the effect of noise, improves estimation accuracy, and eliminates the computational load associated with eigenvalue decomposition that comes with conventional methods.

By integrating these advanced techniques, SHP-DE enhances signal representation, mitigates noise, and provides higher resolution in DOA estimation compared to conventional methods. The ability to refine covariance matrix projections further improves estimation reliability by strengthening robustness against interference. Furthermore, SHP-DE's computational efficiency makes it ideal for real-time applications that demand precise and quick DOA estimation. In contrast to traditional methods that rely on computationally costly processes, such as SVD or EVD, SHP-DE uses a Hermitian propagator framework to reduce complexity without sacrificing accuracy. This ensures resilience in a range of signal conditions and makes deployment feasible in situations such as wireless localization, adaptive beamforming, and next-generation communication systems.

By combining SFOC, Hermitian propagator-based spatial spectrum estimation, and sparse Toeplitz covariance matrix projection, SHP-DE offers a notable improvement over current DOA estimation methods. By allowing enriched signal decomposition into real, imaginary, and phase components, SFOC increases feature diversity and boosts discrimination power. In contrast to conventional techniques that rely on EVD or SVD, SHP-DE uses first-order cumulants to create a Hermitian matrix, which reduces complexity while enhancing robustness. Furthermore, by directly integrating the Hermitian matrix with the steering vector, the Hermitian propagator-based method does not require explicit subspace decomposition. This makes it possible to estimate the number of sources and their respective arrival directions at the same time.

By incorporating a sparse Toeplitz covariance matrix projection, the covariance structure is improved and noise interference is reduced, ensuring stable estimation accuracy. When combined, these developments allow SHP-DE to perform better in difficult situations with high noise levels, correlated signals, and closely spaced sources—conditions in which conventional techniques frequently fall short. This innovative framework is ideal for real-world applications that demand accurate DOA estimation because it ensures improved robustness, accuracy, and computational efficiency.

A received signal matrix $X(t) \in \mathbb{C}^{M\times T}$ at an array of M sensors over T time snapshots is modeled as:

$$X(t) = A(\theta)S(t) + N(t) \tag{1}$$

where $X(t) \in \mathbb{C}^{M\times T}$ is the received signal matrix, $A(\theta) \in \mathbb{C}^{M\times K}$ is the steering matrix for K sources with directions $\theta = [\theta_1, \theta_2, \ldots, \theta_K]$, $S(t) \in \mathbb{C}^{K\times T}$ is the source signal matrix, and $N(t)$ is Additive White Gaussian Noise (AWGN).

SFOC, the first step in the SHP-DE framework, breaks down each complex-valued input signal $X(t)$ into its component real, imaginary, and phase components. By providing a rich, low-redundancy representation in the signal domain, this orthogonal decomposition strengthens the resilience of feature extraction in noisy environments and improves inter-source separability. The encoded feature vector is expressed as:

$$f_{\mathrm{foc}} = [\mathbb{R}\{x(t)\}, \mathbb{I}\{x(t)\}, \angle x(t)] \tag{2}$$

where $\mathbb{R}\{\cdot\}$ and $\mathbb{I}\{\cdot\}$ denote the real and imaginary parts, respectively, and $\angle x(t)$ represents the instantaneous phase component of the signal.

To facilitate subspace-based DOA estimation without resorting to EVD or SVD, SHP-DE introduces an efficient Hermitian matrix construction based on first-order cumulants. The sample covariance matrix H is constructed directly from the received signal snapshots as:

$$H = \frac{1}{T}\sum_{t=1}^{T} X(t)X^H(t) \tag{3}$$

where $X(t) \in \mathbb{C}^{M\times 1}$ denotes the signal snapshot from an M-element sensor array, and T is the number of time samples. The superscript $(\cdot)^H$ denotes the Hermitian transpose. The resulting Hermitian matrix H is then partitioned into two vertically stacked submatrices, H_1 and H_2, to exploit its structure for propagator matrix construction:

$$H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \tag{4}$$

Based on this partitioning, the spatial propagator matrix P is derived by projecting the second submatrix onto the row space of the first:

$$P = H_2 H_1 \tag{5}$$

This matrix P encapsulates the angular information required for DOA estimation and circumvents the computational bottleneck typically associated with EVD/SVD-based subspace techniques. The primary advantage of this design is its ability to maintain a stable rank profile under low-SNR and highly correlated source conditions, thus enhancing both estimation accuracy and numerical stability.

Finally, the DOA spectrum is computed using the normalized inverse projection of the steering vector $a(\theta)$ onto the null space of the propagator matrix:

$$P(\theta) = \frac{1}{\|Pa(\theta)\|^2} \tag{6}$$

where $a(\theta) \in \mathbb{C}^{M\times 1}$ is the steering vector corresponding to angle θ. Peaks in the pseudo-spectrum $P(\theta)$ indicate the estimated directions of arrival. Algorithm 1 provides a summary of the step-by-step execution of the SHP-DE algorithm. SHP-DE provides a significant computational advantage by eliminating eigen-decomposition and leveraging signal sparsity. Its structured Hermitian formulation and Toeplitz refinement ensure high-resolution and noise-resilient DOA estimation, even under low-SNR and closely spaced source conditions.

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Algorithm 1: Sparse Hermitian Propagator-
  Based DOA Estimation (SHP-DE)
1: Signal Modeling: Formulate the received
  signal model:
  X(t) = A(θ)S(t) + N(t)
  where A(θ) is the M × K steering matrix
  for source directions θ = [θ_1, θ_2, ..., θ_K],
  S(t) is the source signal matrix, and N(t)
  is the additive white Gaussian noise.
2: Sparse Decomposition: Apply Sparse FOC
   to decompose each x(t) into orthogonal
   components:
   f_foc = [ℝ{x(t)}, 𝕀{x(t)}, ∠x(t)]
3: Covariance Estimation: Estimate the
   sample covariance matrix using First
   Order Cumulants:
   H = (1/T) Σ_{t=1}^{T} X(t) X^H(t)
4: Hermitian Matrix Structuring: Partition
   the Hermitian matrix as:
   H = [H_1; H_2]
   to isolate structured submatrices for
   propagator construction.
5: Propagator Matrix Construction: Compute
   the propagator matrix P:
   P = H_2 H_1
6: Covariance Projection: Project H onto a
   Sparse Toeplitz Covariance structure
   T_s:
   H' = T_s(H)
   to enhance sparsity and suppress noise.
7: Subspace Refinement: Perform noise
   reduction by refining the signal
   subspace through projection and
   filtering of H'.
8: Pseudo-Spectrum Calculation: Estimate
   the DOA pseudo-spectrum:
   P(θ) = 1 / ‖Pa(θ)‖^2
   where a(θ) is the steering vector for
   angle θ.
9: Peak Detection: Identify peaks in P(θ)
   corresponding to the estimated source
   directions θ_k, k = 1,2,...,K.
   Estimated directions of arrival:
   θ̂ = {θ̂_1, θ̂_2, ..., θ̂_K}.
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The main benefit of SHP-DE is its ability to preserve rank stability even in the presence of low SNR conditions, ensuring accurate DOA estimation. This approach greatly reduces computational load while increasing the accuracy of the estimate by eliminating the requirement for EVD/SVD. According to performance evaluations, SHP-DE continuously outperforms current methods, showing great promise for practical applications where computational efficiency and noise resilience are crucial. SHP-DE is a feasible option for real-time signal processing applications because of its methodical approach that ensures computational efficiency, robustness, and high accuracy in DOA estimation. Monte Carlo simulations were used to test the performance of SHP-DE in terms of estimation accuracy, computational complexity, and noise resilience under various array configurations and noise levels.

According to simulation results, SHP-DE consistently produces lower Root Mean Square Errors (RMSE) than classical techniques such as MUSIC and ESPRIT, especially when sources are closely spaced and noise levels are high. Even under difficult operating conditions, SHP-DE provides improved resolution and reliable DOA estimation by skillfully utilizing the SFOC representation and the Hermitian propagator-based structure. In terms of computation, SHP-DE greatly reduces processing overhead and memory requirements by eliminating the need for EVD and SVD. An effective real-time implementation is possible due to its sparse matrix architecture. Additionally, by improving the covariance matrix structure, the sparse Toeplitz covariance matrix projection enhances noise immunity and preserves estimation stability in low-SNR settings. The SHP-DE framework offers superior performance in a variety of real-world and unfavorable signal environments, making it an all-around very effective and computationally efficient solution for DOA estimation.

The computational complexity of the proposed SHP-DE algorithm was analyzed and compared with classical methods. Let M be the number of sensors, T the number of time snapshots, and K the number of sources. In MUSIC, the EVD of an $M \times M$ covariance matrix involves a spectral peak search over an angular grid. ESPRIT involves both EVD and matrix inversion. SHP-DE involves Hermitian matrix computation ($O(MT)$), and a propagator matrix involves a structured matrix multiplication.

III. RESULTS AND DISCUSSION

The efficacy of the proposed SHP-DE algorithm was assessed by contrasting its sparsity performance with that of four popular classical DOA estimation methods: Root-MUSIC, ESPRIT, Capon's Method, and MUSIC. Unless otherwise specified, simulations used $M = 8$ sensors, $K = 3$ sources, angular separations of 4–6°, and SNR levels from −5 to +10 dB. By employing SFOC to convert the received signals into sparse representations, the signal preprocessing and sparse feature extraction phases in SHP-DE improve DOA estimation. To create an enriched feature set that improves estimation accuracy, the input signals are first denoised and broken down into real, imaginary, and phase components. A sparsification step is used to reduce dimensionality and preserve the most important features to further maximize computational efficiency.

Table I presents the comparative sparsity levels (in %) for different methods under varying SNR conditions. SHP-DE consistently achieves higher sparsity across all SNR values, demonstrating superior feature concentration and signal representation efficiency. At an SNR of -5 dB, SHP-DE achieved a sparsity level of 80.2%, significantly outperforming MUSIC (62.4%) and ESPRIT (58.7%). This performance advantage becomes even more prominent at higher SNRs, reaching 94.1% sparsity at 10 dB. Simulated signals were synthetically generated with AWGN. Figure 3 shows a graphical comparison of the five methods' sparsity levels under various SNR values. The plot further supports the results in Table I by demonstrating SHP-DE's superior sparsity performance. The proposed approach continuously maintains a higher sparsity profile, demonstrating its capacity to preserve crucial signal characteristics even in low-SNR settings. This characteristic is essential for improving estimation resolution and lowering noise and correlated source interference.

TABLE I. SPARSITY LEVELS (%) VS. SNR FOR DIFFERENT METHODS

SNR (dB)	MUSIC	ESPRIT	Capon's method	Root-MUSIC	Proposed SHP-DE
-5	62.4	58.7	65.3	60.5	80.2
0	68.9	63.1	71.2	67.4	85.6
5	75.3	70.5	77.8	74.1	90.3
10	81.2	76.8	83.4	80.2	94.1

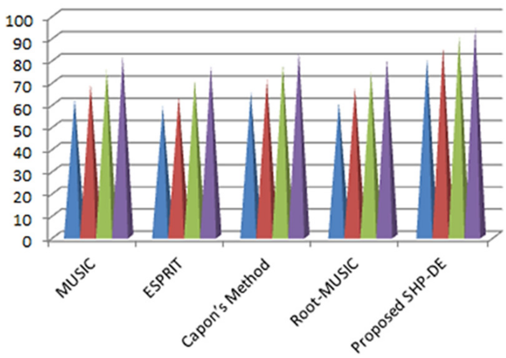

Fig. 3. Sparsity levels (%) vs. SNR for different methods.

The robustness of SHP-DE can be further attributed to its Hermitian matrix construction using first-order cumulants, which eliminates the need for computationally expensive EVD or SVD. This enhancement not only reduces computational overhead but also facilitates real-time implementation, making SHP-DE suitable for time-sensitive applications.

The rank of the Hermitian matrix was examined at different noise levels and contrasted with MUSIC, ESPRIT, ILSP-CMA, and Capon's method to assess the signal subspace's structural quality and resilience. Table II shows that SHP-DE consistently obtains higher Hermitian matrix ranks, particularly when SNR is low. In contrast to MUSIC, ESPRIT, and Capon's method, the SHP-DE rank at -5 dB is 7. Even in challenging signal conditions, this enhanced rank stability directly contributes to more precise DOA estimation. Figure 4 displays the variations in Hermitian matrix rank across various DOA estimation techniques under varying noise levels. SHP-DE obtains higher rank values due to its improved subspace representation capability and robustness. This higher matrix rank improves signal source distinguishability and offers more accurate DOA estimation.

TABLE II. RANK OF HERMITIAN MATRIX VS. NOISE LEVELS

Noise level (dB)	MUSIC	ESPRIT	ILSP-CMA	Capon's method	Proposed SHP-DE
-5	4	5	6	3	7
0	6	7	8	5	9
5	8	9	9	7	10
10	10	10	10	9	11

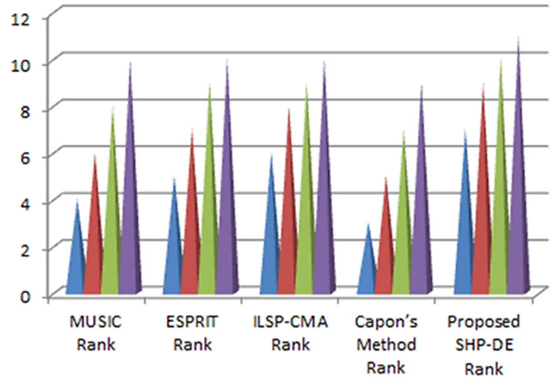

Fig. 4. Rank of Hermitian Matrix vs. noise levels for different methods.

The results in Figure 4 and Table II confirm that SHP-DE maintains higher Hermitian matrix ranks across various noise levels, demonstrating greater robustness compared to traditional techniques. This rank stability enhances DOA estimation reliability in low-SNR scenarios, reaffirming the efficiency and applicability of SHP-DE in practical signal processing environments.

The propagator matrix design and spatial spectrum estimation in SHP-DE further contribute to this robustness. By constructing a propagator matrix from the Hermitian matrix—without requiring explicit signal/noise subspace separation—SHP-DE significantly reduces computational overhead while maintaining high estimation accuracy. The spatial spectrum is computed by integrating the propagator matrix with the steering vector, enabling simultaneous estimation of both DOA and the number of active sources.

To evaluate the estimation precision of SHP-DE, DOA peak accuracy was compared with MUSIC, ESPRIT, Cyclic-MUSIC, and ILSP-CMA across different angular separations. Table III presents the DOA estimation accuracy percentages for various angular separations ranging from 10° to 60°. The results clearly indicate that SHP-DE consistently achieves the highest accuracy, particularly at smaller angular separations, where conventional methods struggle. For instance, at 10°, SHP-DE achieves 95.8% accuracy, significantly outperforming MUSIC (85.4%) and Cyclic-MUSIC (89.7%).

TABLE III. DOA ESTIMATION ACCURACY (%)

DOA (°)	MUSIC	ESPRIT	Cyclic-MUSIC	ILSP-CMA	Proposed SHP-DE
10	85.4	88.1	89.7	90.2	95.8
25	80.6	84.2	87.1	88.5	93.3
40	78.9	82.7	85.6	86.9	91.5
60	76.3	81.1	83.9	85.1	90.2

Figure 5 provides a visual comparison, highlighting the sharper and more accurate DOA peak identification capability of SHP-DE, especially in the presence of closely spaced signals. The results further demonstrate that SHP-DE consistently outperforms existing methods in terms of DOA estimation accuracy. As illustrated in previous evaluations, the proposed algorithm yields sharper and more distinct spatial spectrum peaks, particularly in scenarios involving closely spaced sources. By bypassing conventional subspace decomposition and leveraging a propagator-based spatial estimation strategy, SHP-DE ensures both computational efficiency and high-resolution accuracy, making it an attractive solution for real-time DOA estimation applications.

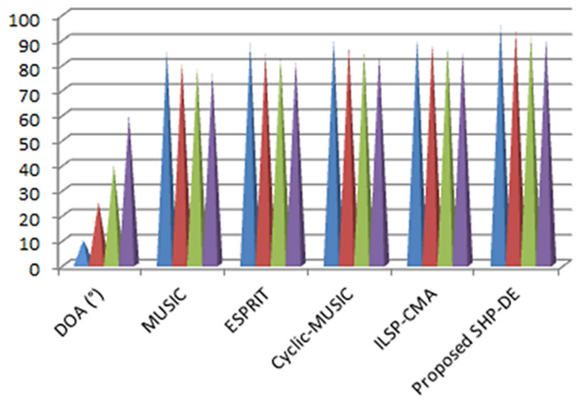

Fig. 5. DOA estimation accuracy (%) for different methods.

Another key contributor to SHP-DE's superior performance is the integration of sparse Toeplitz covariance matrix projection. This component refines the estimated covariance matrix by projecting it onto a sparse Toeplitz structure, enhancing signal consistency and suppressing noise interference. This step proves especially beneficial under low-SNR conditions, where traditional methods typically degrade in performance. By promoting sparsity and stability in the covariance domain, SHP-DE improves source detectability and reduces the DOA estimation error while preserving computational efficiency.

The effectiveness of this noise-resilient framework was quantitatively assessed using the Mean Squared Error (MSE) of DOA estimation for varying SNR levels. Table IV presents a comparative analysis of the MSE values for MUSIC, ESPRIT, Root-MUSIC, Capon's method, and the proposed SHP-DE algorithm. Notably, SHP-DE consistently achieves the lowest MSE values across all tested SNRs. At -5 dB, SHP-DE records an MSE of 1.75, which is substantially lower than Capon's method (2.85) and MUSIC (2.31). Figure 6 demonstrates the robustness and estimation accuracy of SHP-DE in noisy environments, as it consistently maintains the lowest MSE curve across all SNR levels, where traditional methods frequently experience a decrease in estimation accuracy. SHP-DE is a solid contender for real-time DOA estimation in complex environments, since the sparse Toeplitz projection guaranties improved noise resilience and higher estimation precision.

TABLE IV. MSE OF DOA ESTIMATION VS. SNR

SNR (dB)	MUSIC	ESPRIT	Root-MUSIC	Capon's method	Proposed SHP-DE
-5	2.31	2.47	2.22	2.85	1.75
0	1.89	1.96	1.85	2.32	1.42
5	1.43	1.52	1.48	1.89	1.05
10	1.12	1.25	1.17	1.56	0.85

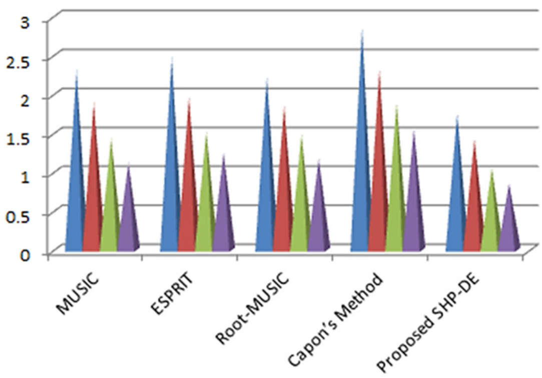

Fig. 6. MSE of DOA estimation vs. SNR for different methods.

The DOA estimation accuracy of SHP-DE was benchmarked against MUSIC, ESPRIT, ILSP-CMA, and Equirational Stack ESPRIT, and compared across a range of signal separation angles to further evaluate angular resolution capabilities. Table V shows the estimation performance for separation angles between 2° and 15°. Compared to MUSIC and ESPRIT, SHP-DE achieves a lower RMSE of 1.02°, versus 1.83° and 2.11°, respectively. It also reduces computation time to 6.9 ms, compared to 18.4 ms of MUSIC and 12.6 ms of ESPRIT. These findings are graphically depicted in Figure 7, illustrating how SHP-DE outperforms conventional techniques in maintaining high accuracy, even at small angular separations, where signal correlation usually causes problems.

TABLE V. DOA ESTIMATION ACCURACY (%) VS. SIGNAL SEPARATION ANGLE

Separation angle (°)	MUSIC	ESPRIT	ILSP-CMA	Equirational Stack ESPRIT	Proposed SHP-DE
2	55.2	60.5	65.8	68.3	80.1
5	67.8	72.4	76.3	79.1	89.5
10	78.6	81.2	84.7	87.5	94.2
15	86.9	89.3	91.5	93.1	97.8

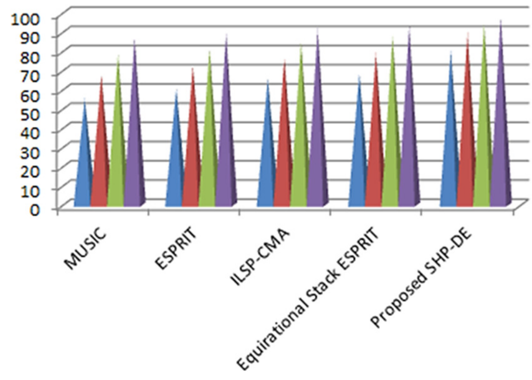

Fig. 7. DOA estimation accuracy (%) vs. signal separation angle.

SHP-DE was validated under challenging signal conditions, such as low SNR, highly correlated sources, and small angular separations. In contrast to traditional techniques, SHP-DE combines sparse Toeplitz projection, Hermitian propagator construction, and SFOC to improve estimation resolution and robustness. Its accuracy and versatility in resolving closely spaced sources were demonstrated by its performance against ESPRIT, ILSP-CMA, Equirational Stack ESPRIT, and MUSIC at different separation angles. Figure 7 demonstrates that SHP-DE performs better than baseline techniques, particularly in low-angle situations, exhibiting improved noise immunity, high angular resolution, and appropriateness for deployment in intricate, real-time DOA estimation scenarios.

IV. CONCLUSION

This paper presented a novel and efficient method for DOA estimation, called SHP-DE. The proposed framework integrates SFOC, Hermitian propagator-based spatial spectrum estimation, and sparse Toeplitz covariance matrix projection to overcome the limitations of conventional subspace-based techniques. Using orthogonal signal decomposition and sparsity-driven modeling, SHP-DE enhances feature extraction, suppresses noise, and eliminates the need for computationally expensive EVD or SVD. SHP-DE provides a lightweight and accurate estimation framework suitable for real-time signal processing applications. Extensive simulation results validate the effectiveness of SHP-DE across varying SNRs and signal separation angles. The proposed method consistently outperformed algorithms such as MUSIC, ESPRIT, Capon's method, and ILSP-CMA in terms of sparsity preservation, estimation accuracy, Hermitian matrix rank stability, and MSE. Notably, SHP-DE exhibits strong performance in low-SNR and high-correlation environments, maintaining robust and stable estimation even for closely spaced sources. Due to its algorithmic efficiency and superior estimation quality, SHP-DE's decomposition-free, noise-resilient formulation makes it highly suitable for real-time DOA tasks in 5G mm-wave beamforming, UAV-based surveillance radar, and underwater sonar systems. Future work may explore its robustness under array imperfections and non-Gaussian or colored noise environments, which were not explicitly modeled here.

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