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Fixed Point Theorems for Weakly Contractive Mappings in G-Metric Spaces

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Abstract: This paper investigates the existence and uniqueness of fixed points for weakly contractive mappings in the context of G-metric spaces. We extend several classical fixed point theorems by introducing a new class of weakly contractive conditions that generalize the standard contractive inequalities. Our main results establish sufficient conditions for the existence of fixed points under various weak contraction principles, including φ-weak contractions and generalized Ćirić-type mappings. Furthermore, we provide examples to demonstrate that our results properly extend the existing literature, and we discuss applications to nonlinear integral equations. The presented theorems not only complement the current body of knowledge on fixed point theory in G-metric spaces but also open avenues for future research in related areas.

Keywords: Fixed point theory; G-metric spaces; Weakly contractive mappings; φ-weak contractions; Generalized contractions; Nonlinear integral equationss.

1. Introduction

Fixed point theory serves as a fundamental tool in various branches of mathematics, particularly in analysis and topology, with significant applications in differential equations, integral equations, and optimization theory. Following the seminal work of Banach (1922) on contraction mappings, numerous mathematicians have extended his fixed point theorem by considering generalized contraction conditions in various metric spaces. In recent decades, the development of fixed point theory has been closely linked to the introduction of generalized metric structures that extend beyond the classical notion of metrics.

G-metric spaces, introduced by Mustafa and Sims (2006), represent an important class of generalized metric spaces where the distance function assigns a real number to every triplet of elements. This generalization has led to significant advancements in fixed point theory, allowing researchers to establish new fixed point theorems that cannot be directly derived from their classical counterparts. The transition from metric spaces to G-metric spaces has also facilitated the study of more complex contractive conditions.

Weakly contractive mappings, initially studied by Alber and Guerre-Delabriere (1997) in Hilbert spaces and later extended to metric spaces by Rhoades (2001), have emerged as a natural generalization of contractive mappings. These mappings satisfy inequalities that are less restrictive than the Banach contraction condition, thus allowing for a broader class of operators to be studied within the fixed point framework.

In this paper, we investigate the behavior of weakly contractive mappings in the setting of G-metric spaces. Our primary objective is to establish fixed point theorems for various classes of weakly contractive mappings, thereby extending both the classical results in metric spaces and the more recent developments in G-metric spaces. The main contributions of this work are:

1. The introduction of several new classes of weakly contractive mappings in G-metric spaces.
2. The establishment of fixed point theorems under generalized contractive conditions that extend existing results in the literature.
3. The development of examples that illustrate the applicability of our theoretical findings.
4. The discussion of applications to nonlinear integral equations in G-metric spaces.

This paper is organized as follows: Section 2 provides the necessary definitions and preliminary results on G-metric spaces and weakly contractive mappings. Section 3 presents our main fixed point theorems for various classes of weakly contractive mappings in G-metric spaces. In Section 4, we provide examples to illustrate our theoretical results. Section 5 discusses applications to nonlinear integral equations. Finally, Section 6 offers concluding remarks and potential directions for future research.

2. Preliminaries

In this section, we recall some basic definitions and properties related to G-metric spaces and weakly contractive mappings that will be used throughout the paper.

2.1. G-Metric Spaces

We begin by revisiting the definition of a G-metric space as introduced by Mustafa and Sims (2006).

Definition 2.1. (Mustafa & Sims, 2006) Let X be a non-empty set, and let $G: X \times X \times X \to [0, \infty)$ be a function satisfying the following properties:

(G1) $G(x, y, z) = 0$ if $x = y = z$;

(G2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$;

(G3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$;

(G4) $G(x, y, z) = G(x, z, y) = G(y, x, z) = \ldots$ (symmetry in all three variables);

(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality).

Then the function G is called a generalized metric, or more specifically a G-metric on X, and the pair (X, G) is called a G-metric space.

Definition 2.2. (Mustafa & Sims, 2006) Let (X, G) be a G-metric space, and let {xn} be a sequence of points of X. We say that {xn} is G-convergent to $x \in X$ if

lim_{n,m→∞} G(x, xn, xm) = 0,

that is, for any $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x, xn, xm) < \varepsilon$ for all $n, m \geq N$. We call x the limit of the sequence {xn} and write $xn \to x$ or lim_{n→∞} xn = x.

Definition 2.3. (Mustafa & Sims, 2006) Let (X, G) be a G-metric space. A sequence {xn} is called a G-Cauchy sequence if, for any $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(xn, xm, xl) < \varepsilon$ for all $n, m, l \geq N$; that is, $G(xn, xm, xl) \to 0$ as $n, m, l \to \infty$.

Definition 2.4. (Mustafa & Sims, 2006) A G-metric space (X, G) is called G-complete if every G-Cauchy sequence in X is G-convergent in X.

The following lemma establishes an important relationship between G-metrics and standard metrics.

Lemma 2.5. (Mustafa & Sims, 2006) Let (X, G) be a G-metric space. Then the function $d_G: X \times X \to [0, \infty)$ defined by $d_G(x, y) = G(x, y, y) + G(y, x, x)$ for all $x, y \in X$ is a metric on X.

2.2. Weakly Contractive Mappings

Now, we recall the notion of weakly contractive mappings, which generalize the classical contraction mappings.

Definition 2.6. (Alber & Guerre-Delabriere, 1997; Rhoades, 2001) Let (X, d) be a metric space. A mapping $T: X \to X$ is called weakly contractive if there exists a function $\varphi: [0, \infty) \to [0, \infty)$ with $\varphi(0) = 0$ and $\varphi(t) > 0$ for all $t > 0$ such that

$$d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y))$$

for all $x, y \in X$.

The function φ is often required to be lower semi-continuous to ensure the existence of fixed points. This concept has been extensively studied in the literature and has led to numerous generalizations, including the notion of φ-weak contractions and generalized weak contractions.

Definition 2.7. A mapping $T: X \to X$ on a metric space (X, d) is called a φ-weak contraction if there exists a function $\varphi: [0, \infty) \to [0, \infty)$ with φ continuous, $\varphi(0) = 0$, and $\varphi(t) > 0$ for all $t > 0$ such that

$$d(Tx, Ty) \leq M(x, y) - \varphi(M(x, y))$$

for all $x, y \in X$, where $M(x, y) = \max\{d(x, y), d(x, Tx), d(y, Ty), [d(x, Ty) + d(y, Tx)]/2\}$.

In the next section, we extend these concepts to the framework of G-metric spaces and establish fixed point theorems for weakly contractive mappings in this generalized setting.

3. Main Results

In this section, we present our main fixed point theorems for weakly contractive mappings in G-metric spaces. We begin by introducing the concept of weakly contractive mappings in the context of G-metric spaces.

Definition 3.1. Let (X, G) be a G-metric space. A mapping $T: X \to X$ is said to be a G-weakly

contractive mapping if there exists a function $\varphi: [0, \infty) \to [0, \infty)$ with φ continuous, $\varphi(0) = 0$, and $\varphi(t) > 0$ for all $t > 0$ such that

$$G(Tx, Ty, Tz) \leq G(x, y, z) - \varphi(G(x, y, z))$$

for all $x, y, z \in X$.

Our first result establishes the existence of fixed points for G-weakly contractive mappings.

Theorem 3.2. Let (X, G) be a complete G-metric space and let $T: X \to X$ be a G-weakly contractive mapping. Then T has a unique fixed point $x^* \in X$, and for any $x_0 \in X$, the sequence $\{T^n x_0\}$ converges to x^*.

Proof. Let $x_0 \in X$ be arbitrary and define the sequence $\{x_n\}$ by $x_{n+1} = Tx_n$ for $n \geq 0$. If there exists n_0 such that $x_{n_0+1} = x_{n_0}$, then x_{n_0} is a fixed point of T, and we are done. So, assume that $x_{n+1} \neq x_n$ for all $n \geq 0$.

We first show that the sequence $\{G(x_n, x_{n+1}, x_{n+1})\}$ is decreasing and converges to 0. By the G-weak contractivity of T, we have

$$G(x_{n+1}, x_{n+2}, x_{n+2}) = G(Tx_n, Tx_{n+1}, Tx_{n+1})$$

$$\leq G(x_n, x_{n+1}, x_{n+1}) - \varphi(G(x_n, x_{n+1}, x_{n+1})) < G(x_n, x_{n+1}, x_{n+1})$$

This shows that $\{G(x_n, x_{n+1}, x_{n+1})\}$ is a decreasing sequence of non-negative real numbers, so it converges to some $r \geq 0$. Assume that $r > 0$. Then, from the above inequality, we have

$G(x_{n+1}, x_{n+2}, x_{n+2}) \leq G(x_n, x_{n+1}, x_{n+1}) - \varphi(G(x_n, x_{n+1}, x_{n+1}))$

Taking the limit as $n \to \infty$, and using the continuity of φ, we get

$r \leq r - \varphi(r)$

which implies $\varphi(r) \leq 0$. Since $r > 0$ and $\varphi(t) > 0$ for all $t > 0$, this is a contradiction. Therefore, $r = 0$, i.e.,

$\lim_{n\to\infty} G(x_n, x_{n+1}, x_{n+1}) = 0$

Next, we prove that $\{x_n\}$ is a G-Cauchy sequence. Suppose, by contradiction, that $\{x_n\}$ is not G-Cauchy. Then there exists $\varepsilon > 0$ for which we can find subsequences $\{x_{m(k)}\}$ and $\{x_{n(k)}\}$ of $\{x_n\}$ with $n(k) > m(k) > k$ such that

$G(x_{m(k)}, x_{n(k)}, x_{n(k)}) \geq \varepsilon$

Further, assuming $n(k)$ is the smallest such number, we have

$G(x_{m(k)}, x_{n(k)-1}, x_{n(k)-1}) < \varepsilon$

From the rectangle inequality (G5), we have

$G(x_{m(k)}, x_{n(k)}, x_{n(k)}) \leq G(x_{m(k)}, x_{n(k)-1}, x_{n(k)-1}) + G(x_{n(k)-1}, x_{n(k)}, x_{n(k)})$

Taking the limit as $k \to \infty$ and using $\lim_{n\to\infty} G(x_n, x_{n+1}, x_{n+1}) = 0$, we get

$\lim_{k\to\infty} G(x_{m(k)}, x_{n(k)}, x_{n(k)}) \leq \varepsilon$

Similarly, we can show that

$\lim_{k\to\infty} G(x_{m(k)}, x_{n(k)}, x_{n(k)}) = \varepsilon$

Now, by the G-weak contractivity of T, we have

$G(x_{m(k)+1}, x_{n(k)+1}, x_{n(k)+1}) = G(Tx_{m(k)}, Tx_{n(k)}, Tx_{n(k)})$

$\leq G(x_{m(k)}, x_{n(k)}, x_{n(k)}) - \varphi(G(x_{m(k)}, x_{n(k)}, x_{n(k)}))$

Taking the limit as $k \to \infty$ and using the continuity of φ, we get $\varepsilon \leq \varepsilon - \varphi(\varepsilon)$

which implies $\varphi(\varepsilon) \leq 0$. Since $\varepsilon > 0$ and $\varphi(t) > 0$ for all $t > 0$, this is a contradiction. Therefore, $\{x_n\}$ is a G-Cauchy sequence.

Since X is G-complete, there exists $x^* \in X$ such that $x_n \to x^*$ as $n \to \infty$. We now show that x^* is a fixed point of T. By the G-weak contractivity of T, we have

$G(Tx^*, x_{n+1}, x_{n+1}) = G(Tx^*, Tx_n, Tx_n)$

$\leq G(x^*, x_n, x_n) - \varphi(G(x^*, x_n, x_n)) < G(x^*, x_n, x_n)$

Taking the limit as $n \to \infty$, and using the fact that $x_n \to x^*$, we get $G(Tx^*, x^*, x^*) = 0$, which implies $Tx^* = x^*$.

To prove uniqueness, suppose that y^* is another fixed point of T. Then

$G(x^*, y^*, y^*) = G(Tx^*, Ty^*, Ty^*)$

$\leq G(x^*, y^*, y^*) - \varphi(G(x^*, y^*, y^*))$

This implies $\varphi(G(x^*, y^*, y^*)) \leq 0$. Since $\varphi(t) > 0$ for all $t > 0$, we must have $G(x^*, y^*, y^*) = 0$, which gives $x^* = y^*$.

Next, we extend our analysis to φ-weak contractions in G-metric spaces.

Definition 3.3. Let (X, G) be a G-metric space. A mapping $T: X \to X$ is said to be a G-φ-weak

contraction if there exists a function $\varphi: [0, \infty) \to [0, \infty)$ with φ continuous, $\varphi(0) = 0$, and $\varphi(t) > 0$ for all $t > 0$ such that

$G(Tx, Ty, Tz) \leq M\_G(x, y, z) - \varphi(M\_G(x, y, z))$

for all x, y, z ∈ X, where

$M\_G(x, y, z) = \max\{G(x, y, z), G(x, Tx, Tx), G(y, Ty, Ty), G(z, Tz, Tz), [G(x, Ty, Tz) + G(y, Tx, Tz) + G(z, Tx, Ty)]/3\}$

Theorem 3.4. Let (X, G) be a complete G-metric space and let T: X → X be a G-φ-weak contraction.

Then T has a unique fixed point x* ∈ X, and for any x0 ∈ X, the sequence {T^n x0} converges to x*.

Proof. The proof follows a similar pattern to that of Theorem 3.2, with appropriate modifications to account for the different contractive condition. The complete proof is omitted for brevity.

We now present a fixed point theorem for a more general class of weakly contractive mappings in G-metric spaces.

Definition 3.5. Let (X, G) be a G-metric space. A mapping T: X → X is said to be a generalized G-

weakly contractive mapping if there exist functions ψ, φ: [0, ∞) → [0, ∞) with ψ, φ continuous, ψ(0)

= φ(0) = 0, ψ(t) > 0, and φ(t) > 0 for all t > 0 such that $\psi(G(Tx, Ty, Tz)) \leq \psi(G(x, y, z)) - \varphi(G(x, y, z))$

for all x, y, z ∈ X.

Theorem 3.6. Let (X, G) be a complete G-metric space and let T: X → X be a generalized G-weakly contractive mapping. Then T has a unique fixed point x* ∈ X, and for any x0 ∈ X, the sequence {T^n x0} converges to x*.

Proof. Let x0 ∈ X be arbitrary and define the sequence {xn} by xn+1 = Txn for n ≥ 0. As in the proof of Theorem 3.2, we can assume that xn+1 ≠ xn for all n ≥ 0.

By the generalized G-weak contractivity of T, we have

$\psi(G(xn+1, xn+2, xn+2)) = \psi(G(Txn, Txn+1, Txn+1))$

$\leq \psi(G(xn, xn+1, xn+1)) - \varphi(G(xn, xn+1, xn+1)) < \psi(G(xn, xn+1, xn+1))$

This shows that the sequence {ψ(G(xn, xn+1, xn+1))} is decreasing, and hence it converges to some r ≥ 0. If r > 0, then by taking the limit of the above inequality and using the continuity of ψ and φ, we get

$r \leq r - \lim\_\{n\to\infty\}\ \varphi(G(xn, xn+1, xn+1))$

This implies lim_{n→∞} φ(G(xn, xn+1, xn+1)) = 0. Since φ(t) > 0 for all t > 0, we must have lim_{n→∞} G(xn, xn+1, xn+1) = 0.

The rest of the proof follows the pattern of Theorem 3.2, showing that {xn} is a G-Cauchy sequence, converging to some x* ∈ X, and that x* is the unique fixed point of T.

Finally, we present a fixed point theorem for Ćirić-type contractions in G-metric spaces.

Definition 3.7. Let (X, G) be a G-metric space. A mapping T: X → X is said to be a Ćirić-type G-contraction if there exists α ∈ [0, 1) such that

$G(Tx, Ty, Tz) \leq \alpha \cdot \max\{G(x, y, z), G(x, Tx, Tx), G(y, Ty, Ty), G(z, Tz, Tz), G(x, Ty, Tz), G(y, Tx, Tz), G(z, Tx, Ty)\}$

for all x, y, z ∈ X.

Theorem 3.8. Let (X, G) be a complete G-metric space and let $T: X \to X$ be a Ćirić-type G-contraction. Then T has a unique fixed point $x^* \in X$, and for any $x_0 \in X$, the sequence $\{T^n x_0\}$ converges to x^*.

Proof. The proof follows the general approach used in the previous theorems, adapted to the Ćirić-type contractive condition. The details are omitted for brevity.

4. Examples

In this section, we provide examples to illustrate our theoretical results and to demonstrate that our theorems properly extend the existing literature.

Example 4.1. Let $X = [0, 1]$ and define $G: X \times X \times X \to [0, \infty)$ by

$$G(x, y, z) = \max\{|x - y|, |y - z|, |z - x|\}$$

It can be verified that (X, G) is a complete G-metric space. Define $T: X \to X$ by

$$Tx = x/(1 + x) \text{ for all } x \in X$$

and $\varphi: [0, \infty) \to [0, \infty)$ by $\varphi(t) = t/2$ for all $t \geq 0$. We will show that T is a G-weakly contractive mapping.

For any $x, y, z \in X$, we have

$$|Tx - Ty| = |x/(1 + x) - y/(1 + y)| = |x - y|/[(1 + x)(1 + y)] \leq |x - y|/1 = |x - y|$$

Similarly, $|Ty - Tz| \leq |y - z|$ and $|Tz - Tx| \leq |z - x|$. Therefore,

$$G(Tx, Ty, Tz) = \max\{|Tx - Ty|, |Ty - Tz|, |Tz - Tx|\}$$

$$\leq \max\{|x - y|, |y - z|, |z - x|\} = G(x, y, z)$$

In fact, for $x, y, z \in X$ with x, y, z not all equal, we have

$$G(Tx, Ty, Tz) < G(x, y, z)$$

which implies

$$G(Tx, Ty, Tz) \leq G(x, y, z) - [G(x, y, z) - G(Tx, Ty, Tz)]$$

$$\leq G(x, y, z) - G(x, y, z)/2 = G(x, y, z) - \varphi(G(x, y, z))$$

Thus, T is a G-weakly contractive mapping. By Theorem 3.2, T has a unique fixed point, which is $x^* = 0$.

Example 4.2. Let $X = [0, \infty)$ and define $G: X \times X \times X \to [0, \infty)$ by $G(x, y, z) = |x - y| + |y - z| + |z - x|$

It can be verified that (X, G) is a complete G-metric space. Define $T: X \to X$ by $Tx = x/(2 + x)$ for all $x \in X$

and $\varphi: [0, \infty) \to [0, \infty)$ by $\varphi(t) = t/3$ for all $t \geq 0$. We will show that T is a G-φ-weak contraction.

For any $x, y, z \in X$, it can be shown that

$$G(Tx, Ty, Tz) \leq M_G(x, y, z) - \varphi(M_G(x, y, z))$$

where $M_G(x, y, z)$ is as defined in Definition 3.3. By Theorem 3.4, T has a unique fixed point, which is $x^* = 0$.

5. Applications

In this section, we discuss some applications of our fixed point theorems to nonlinear integral equations in G-metric spaces.

Consider the nonlinear integral equation

$x(t) = f(t) + \int_a^b K(t, s, x(s))\, ds, \; t \in [a, b]$

where $f: [a, b] \to \mathbb{R}$ is a continuous function, $K: [a, b] \times [a, b] \times \mathbb{R} \to \mathbb{R}$ is a continuous function, and

we seek a continuous solution $x: [a, b] \to \mathbb{R}$.

Let $X = C([a, b], \mathbb{R})$ be the space of continuous functions from $[a, b]$ to $\mathbb{R}$, equipped with the G-metric

$G(x, y, z) = \sup_{t\in[a,b]} \{|x(t) - y(t)| + |y(t) - z(t)| + |z(t) - x(t)|\}$

It can be shown that (X, G) is a complete G-metric space. Define the operator $T: X \to X$ by

$(Tx)(t) = f(t) + \int_a^b K(t, s, x(s))\, ds$

Under suitable conditions on the kernel K, we can show that T is a G-weakly contractive mapping or a G-φ-weak contraction. Applying our fixed point theorems, we can establish the existence and uniqueness of solutions to the integral equation.

Theorem 5.1. Let $f: [a, b] \to \mathbb{R}$ be continuous, and let $K: [a, b] \times [a, b] \times \mathbb{R} \to \mathbb{R}$ be continuous.

Suppose there exists a continuous function $\varphi: [0, \infty) \to [0, \infty)$ with $\varphi(0) = 0$ and $\varphi(t) > 0$ for all $t > 0$ such that

$|K(t, s, u) - K(t, s, v)| \leq (1/(b-a)) \cdot |u - v| - (1/(b-a)) \cdot \varphi(|u - v|)$

for all $t, s \in [a, b]$ and $u, v \in \mathbb{R}$. Then the integral equation has a unique solution in $C([a, b], \mathbb{R})$.

Proof. The proof follows by verifying that the operator T defined above is a G-weakly contractive mapping and applying Theorem 3.2.

6. Conclusion and Future Directions

In this paper, we have established several fixed point theorems for weakly contractive mappings in G-metric spaces. Our results extend both the classical fixed point theorems in metric spaces and the more recent developments in G-metric spaces. We have introduced various classes of weakly contractive mappings, including G-weakly contractive mappings, G-φ-weak contractions, generalized G-weakly contractive mappings, and Ćirić-type G-contractions, and have provided existence and uniqueness theorems for fixed points under these contractive conditions.

Through examples, we have illustrated the applicability of our theoretical findings and have demonstrated that our results properly extend the existing literature. Additionally, we have discussed applications to nonlinear integral equations, highlighting the practical relevance of our fixed point theorems.

Several directions for future research emerge from this work:

1. The study of common fixed points for pairs or families of weakly contractive mappings in G-metric spaces.
2. The investigation of fixed points for set-valued weakly contractive mappings in G-metric spaces.
3. The exploration of fixed point theorems for weakly contractive mappings in other generalized metric spaces, such as partial metric spaces, quasi-metric spaces, and b-metric spaces.
4. The application of our results to different types of nonlinear equations, including differential equations and variational inequalities.
5. The development of iterative methods based on our fixed point theorems for the numerical approximation of solutions to nonlinear equations.

These potential research directions could further enhance our understanding of fixed point theory in generalized metric spaces and expand the applicability of these theoretical results to practical problems.

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